

THESIS ABSTRACT

A Fractal Phase Calculus for Recursive Architectonic Computability

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This thesis presents a fractal phase calculus that contributes to the advancement of computational emulation of human behavior. Previous disparities between theoretical foundations of computability and behavioral complexity are coordinated into alignment. Gödel's theorems, rooted in fractal properties of natural, developmental stages of evolving complexity, describe downwardly assimilated properties of human experience. Tarski recognized Gödel's theorems are not limited to mathematics, but the relationship between mathematics, computation, and human behavior could not be made without the recent identification of the fractal nature of measurable behavioral transition dynamics. One conclusion drawn here is that Gödel's theorems and Church-Turing computability should no longer be described just as properties of mathematics and logic, but of human behavior in general.

This fractal phase calculus unifies these seemingly disparate domains. In doing so, the lack of integral architectonic models for unified semantic architectures and operational processes that emulate human behavior in computational environments is addressed with two novel integral architectonic models of qualia abstraction (architecture) and phase complexity (process). When synthesized as a recursive architectonic they account for the broadest possible range of human experience and behavior. Recursive architectonic properties universal to human experience are captured

and made computable through the fractal phase calculus. In addition to presenting its axioms, the fractal phase calculus is demonstrated by applying it to the development of a molecule to show that these universal architectures and processes underpin not just humans, but entities of all scales that humans conceive. This research was motivated by the needed functions of recursive architectonic software to facilitate a new means for the knowledge creation process. This current contribution has implications for (a) disseminating information, (b) unifying dissemination feedback, (c) supporting the integration and synthesis of differentiation across scales and breadths of participation, (d) identifying uncharted knowledge frontiers, (e) modeling dynamic knowledge, (f) empowering communication, and (g) the creation of a globally participatory theory of everything.

A Fractal Phase Calculus for Recursive Architectonic Computability

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by

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CHAPTER I

INTRODUCTION

In this introduction, I give a general overview of this thesis project about a fractal phase calculus for recursive architectonic computability meant to enable emulation of natural human action in computational environments. This overview includes the objective, contexts, motivations, theoretical foundations, expected and intended audiences, followed by a summary, then terms and definitions.

The Objective

The overarching purpose of my thesis is the development, organization and presentation of a contribution to information science. To make that contribution has required a processual approach: The work in this thesis builds on my previous work and lays foundations for continuing the project post-thesis.

The objective of this current research is to develop a hypothetical model that I am calling fractal phase calculus (FPC) and provide an initial illustration in pseudocode, an outline of the process for future coding. FPC is a hypothetical computational model that organizes elements, elements' relations, and results of those relations using universal abstract principles of the highest orders of complexity currently known. The objective is to create recursively architectonic computational processes that emulate natural human action in computational environments. Such environments can then provide synchronous frameworks between human and machine that unify knowledge building and information

organizing across social scales and knowledge domains. In other words, my aim is to translate very high level abstractions that govern current and future information evolution, into computer-usable language for software.

That software will use my previously-developed model, the spectrum of human imagination (SHIM) as a partial starting point for the recursive architectonic computational architectures and processes. To introduce this objective, I begin with some human characteristics, move to identifying human information needs, and then relate both of these to the information science and the eventual software contexts. But I only briefly contextualize the FPC hypothesis here, because it is explained more thoroughly later after its background, SHIM, is presented in the next chapter.

Humans belong to the species *Homo sapiens*, and as such, share a common anatomical makeup. This anatomical makeup predicates (a) experience and knowledge, (b) the basis for which experience and knowledge are generated, and (c) the way that experience and knowledge are used. By identifying highly abstract characteristics intrinsic to human experience and knowledge, one can observe (a) the architecture and process of the production of experience and knowledge, (b) the basis for which differences occur in this production, and (c) why the outcomes of their applications are as they are. By identifying such patterns of architecture and process of experience and knowledge, the means to express them computationally for the benefit of many becomes possible.

Adult development research indicates that a tiny percentage of people in the world operate with a level of complexity capable of understanding and applying such universal architectures and processes (Kegan & Lahey, 2009; Torbert & Associates, 2004). For

recursive architectonic software to meet the needs of knowledge building and information organization, environments must be made, tools must be built, and support must be given in order for such functionalities to be downwardly assimilated and be directly useable by people who lack the time, expertise, and or competency to produce such high magnitudes of abstractions themselves and operate on them at such high orders of complexity.

Software environments must be easy to use and tools must scale up and down the ranges of complexity inherent in user behavior so that software meets the complexity of users, yielding broad user operability. Thus, knowledge building, information organization, and behavioral support are essential to the unique circumstances and interests from one individual and or group to another and the relationships to be had among them – from the most basic tasks to the most complex innovations. I aspire the work done through this thesis will contribute to such environments, tools, and support for the variety of interests that people have towards understanding and doing things, from the individual to the global level, and everything between.

Context of the Thesis and Motivation

As the foregoing section indicates, the result of doing this thesis research will play a vital role in realizing a longer term vision. That vision drove my previous work, which made the context for this thesis possible: this thesis project belongs to a greater scope of a project that I have been working on over a period of 16 years. That project's goal was to understand the architectures and processes for how everything worked, understand what things were lacking that were needed in the world for it to become a better place, and learn from the mistakes of past and present generations in such efforts so that I might contribute a lasting positive impact on the world.

I went through many epochs of development, leading to the creation of what I call an architectonic. There are different definitions of what an architectonic is, and I mean to use and advance the term in the context of Immanuel Kant's use. Kant (1781/2004) stated of architectonics:

Human reason is by nature architectonic. That is to say, it regards all cognitions as parts of a possible system, and hence accepts only such principles as at least do not incapacitate a cognition to which we may have attained from being placed along with others in a general system. (p. 284)

My definition is consistent with and expands Kant's concept: I define a *standard architectonic* as a model that accounts for, describes, and organizes knowledge about universal principles and or categories in a given scope. Note here that the concept of an architectonic is used in the broadest possible context, and does not inherit its meaning from the architectural field's context of building design. The definition can, however, apply to that architectural context, insofar as architects follow organized principles shared across the field in the creation of their building designs.

I extend the variety of architectonics further by coining and defining two additional types. I define an *integral architectonic* as a model that accounts for, describes, and organizes universal principles and or categories across all domains of knowledge and experience of the human species. The next concept continues and builds on the previous: I define a *recursive architectonic* as a model that (a) accounts for, describes, and organizes universal principles and or categories across all domains of knowledge and experience of the human species and does so in such a way that (b) all the parts are continually informing the whole so that the very universal principles and or

categories themselves and the entire architecture built from them, are in an iterative process of regeneration and improvement by means of the total spectrum of all the developments of all the parts of the whole. Such a transformative dynamic of system-wide recursivity is consistent with what is described empirically and theoretically in neuroscience (see Alexander & Globus, 1996). Based on analyses in my long term projects, I hypothesize that the recursive, iterative transformations just described mirror humanity's developmental dynamics of knowledge and action.

My previous work, then, was devising a model I call the spectrum of human imagination model (SHIM), discussed in Chapter II, which is an integral architectonic. The FPC is a hypothetical mathematical way to represent the recursive architectonic process. This thesis research will contribute to SHIM potentially becoming a recursive architectonic through FPC.

The source that preceded, proceeds, and will continue to motivate me, is compassion from taking the perspective of others. I don't want people to suffer, and I feel a firm obligation to help stop suffering at every level. I want everyone to live healthy, enjoyable, and the longest possible lives. This compels me to help contribute to environments, tools, and support towards peace from the local to the global level. It is my intention that the creation and implementation of this work will be used for these ends.

Theoretical Foundations

The central theoretical foundation for developing the FPC in this thesis research is my integral architectonic, SHIM. SHIM warrants a separate introduction, which is the purpose of Chapter II. As stated earlier, I hypothesize that the recursive, iterative

transformations of the hypothetical recursive architectonic mirror developmental dynamics. Therefore, a primary theoretical foundation with which I undertook this thesis was the developmental transition processes such as those described as stage transitions (Commons & Richards, 2002), fractal transitions (Ross, 2008a), integral process theory (Ross, 2005), and skill theory (Fischer, 1980). Additional investigations during my literature review indicated additional theoretical foundations to help ensure the universality of FPC. I divided into components the further foundations in extant theory I investigated. Those components are introduced next.

The first component to build theoretical foundations was epistemology (Bostock, 2009; Colyvan, 2012) and foundations (Eves, 1990; Mustoe & Barry, 1998) of mathematics. This focus was for insight into the philosophical and mathematical implications of the FPC hypothesis. The second component to build theoretical foundations concerned propositional calculus (Cori & Lascar, 1993/2000; Goldrei, 2005) and fractal mathematics (Baleanu, Diethelm, Scalas, & Trujillo, 2012; Edgar, 2008). These, among others, were to gain insight into converting FPC concepts into mathematical notations. The third component concerned computational informatics such as computational semiotics (Gudwin, 1999) and ontological engineering (Mizoguchi, 2001). These would indicate where and how environments, tools, and support for social development are lacking. Theoretical foundations of this current work, then, are distributed across a wide range of fields, each contributing different bases, insights, and components.

Intended Audiences

The results of this research may gather interest from audiences interested in the effort to develop a hypothesis for modeling all architectures and processes in a unified, recursive way. I hope this FPC research may have implications for and be found useful in informatics and ontology in the information science sense, as it is an attempt at finding and implementing the most efficient and effective means of organizing information. Further, and extending from this, this research may also have implications for the creation of artificial intelligence, in that it may speak to the fundamental architectures and processes that must necessarily be replicated in AI programs to reflect human intelligence.

Audiences of qualitative, quantitative, and mixed method research may find this useful for at least two reasons. Firstly, FPC research may contribute insight for how to ontologically represent said methods of research and the data that are produced by means of those methods. Secondly, the results of this thesis may lead to and or enable an efficient and unified means for correspondence and coordination of ontologies and methodologies and their products within and across them by means of the unified and recursive modeling. This carries over into interdisciplinary and transdisciplinary inquiry as well, because interdisciplinarity represents the effort to bind together disparate disciplines and transdisciplinary represents the effort to unify them (Repko, 2012). In the same vein, this FPC research may have implications for and be found useful in decision science in that FPC research may provide insight into a clearer ontological representation of decision-making processes and the methodologies used to capture them.

Deployed in applications, it builds naturally into policy making for which decision-making processes are necessary at all scales.

When taken in the context of SHIM, the FPC may be of interest to audiences who take interest in neurophysiological and neuroimaging research of semantic categories and fractal dynamics, because whereas SHIM corresponds to universal categories of human knowledge and experience, FPC research seeks to describe the fractal dynamics of the construction of such semantic categories. FPC in conjunction with SHIM may contribute to bridge phenomenological data and neuroscientific data.

Summary

To summarize, the research and product of this thesis project is for the purpose of designing and testing a hypothetical FPC. The goal is to contribute such a model to environments, tools, and support for the variety of interests that people have towards understanding and doing things, from the individual to the global level and everything between.

Terms and Definitions

Major terms used throughout the thesis are defined below. SHIM is introduced in Chapter II, and its terms are deployed in illustrations and in remaining discussions. I have judged it best to omit SHIM terms from this initial list because their relevance will be evident only once SHIM has been reviewed.

Architectonic: A model that accounts for, describes, and organizes knowledge about universal principles and or categories that govern architecture in a given domain.

Integral architectonic: A model that accounts for, describes, and organizes knowledge about universal principles and or categories across all domains of knowledge and experience of the human species.

Recursive architectonic: A model that accounts for, describes, and organizes universal principles and or categories across all domains of knowledge and experience of the human species in such a way that the parts are continually informing the whole so that the very universal principles and or categories themselves and the entire architecture built from it, are in an iterative process of regeneration and improvement by means of the total spectrum of all the developments of all the parts of the whole.

Qualia: Plural of quale, qualia are attributes of experience produced by neurobiological perturbations.

Qualia magnitude: n-dimensional fractal measurement of the tiers (scales) of qualia that are produced from two or more quale

Qualia assembly: any process by which attributes of experience convene into a greater admixture and or magnitude.

Phase order: similar to the definition of order of hierarchical complexity defined as [linear] action in terms of two or more lower-order [linear] actions (Commons, Goodheart, Pekker, Dawson, Draney, & Adams, 2007; Commons, Trudeau, Stein, Richards, & Krause, 1998), order of phase complexity is defined as both linear *and* nonlinear action defined in terms of two or more lower-order and or diagonal actions. Phase orders also differ from orders of hierarchical complexity because phase orders are universalized to account for actions of all entities, not just machines, organisms, and social groups.

Phase transition: an action performed by and between entities that may or may not lead to a more complex order of phase complexity. Dependent upon conditions, the phase transitions from an initial state of static equilibrium are the following: (a) static phase transition, (b) dynamic phase transition, (c) multinamic phase transition (Barker, in preparation).

Multinamism: The simultaneity of static and dynamic properties.

CHAPTER II

THE SPECTRUM OF HUMAN IMAGINATION MODEL

The purpose of this chapter is to give a general overview to situate the spectrum of human imagination model (SHIM) as a background of this thesis project. This section therefore covers the topics in only a very compressed way. The first section discusses the homogeneity of *Homo sapiens* as preexisting theoretical grounds for architectonics as a valid field of inquiry. The second section describes the conception of SHIM rendered as an integral architectonic in transition to become a recursive architectonic, for which this thesis project is a continuation. A crucial adjunct to this section is the Appendix, which presents the current model's organization of tiered categories of qualia abstraction. The third section makes clear the relationship between SHIM and fractal phase calculus (FPC). I give an example of using SHIM in Chapter V.

The Homogeneity of *Homo sapiens*

The purpose of this discussion of homogeneity of *Homo sapiens* is to provide an empirical basis for the validity of architectonics as a scientific endeavor. There are several genetic anthropological models that attempt to explain how modern *Homo sapiens* came to be today, such as the African Replacement Model, the Multiregional Evolution Model, and the Assimilation Model which is a synthesis of the two aforementioned (see Aiello, 1993 and Stringer, 2001). Across these models and their

variations, the modern species within the genus *Homo* are evolutionarily suspected to have become widely genetically homogeneous between 200,000 and 50,000 years ago. For many strictly creationist perspectives, *Homo sapiens* are thought to have existed only between 50,000 to 7,000 years ago. Despite whether a person takes the evolutionary perspective, the creationist perspective, or something between, the heterogeneity of *Homo sapiens* DNA variation across populations has been shown to be very small at 5% or less, though genetic variation could be more (Levy, Sutton, Ng, Feuk, Halpern, Walenz, et al., 2007; The International HapMap 3 Consortium, 2010).

From a human evolutionary genetics point of view, though *Sapiens* is the only remaining species member of the genus *Homo*, and with low levels of genetic diversity among population groups, it remains unclear to what extent that evolutionary forces of mutation, natural selection, and genetic drift will occur within and between populations into the future (Relethford & Harding, 2001). With the additional factor of *Homo sapiens* engineering the genetics in organisms other than itself, and the potentially beneficial and or catastrophic consequences of *Homo sapiens* engineering their own genes, it furthers the uncertainty about the future of what exactly descendent humans will be like (see Commons-Miller, Commons, & Commons, 2008).

From an embryological and ontogenetic point of view, from birth *Homo sapiens* develop biologically in a universally shared way that is common to all of the species (see Blechschmidt, 2004). From a lifespan developmental and gerontological point of view, after birth, through childhood, adulthood, and into old age, commonality of biological change is observed throughout the life of *Homo sapiens* (see Santrock, 2004 and Moody, 2010). The comparison of organismic components of biological similarity of *Homo*

sapiens across lifespan, races, and both genders is commonly known as human anatomy (see Tortora & Derrickson, 2012). The study of abnormal functioning of *Homo sapiens* is called pathology (see Rubin & Strayer, 2012). Despite the approximate 5% genetic variation aforementioned, the emergent anatomical morphology of *Homo sapiens* has room for plenty of diversity before abnormality diverges too far and is deemed pathological and unhealthy. Nonetheless, the overwhelming homogeneity of human anatomy allows knowledge of human anatomy and pathology to exist that is reliable enough to be taught as foundational knowledge for healthcare at accredited universities and irrevocably relied upon for both the scientific advancement of healthcare and by professional healthcare providers alike. In our era, reliability of knowledge regarding human anatomy and pathology allows organizations such as the World Medical Association and World Health Organization to epidemiologically confront pathology worldwide. To summarize, our species relies on the scientific discoveries of homogeneous properties of human physiology to maintain public health.

From a psychophysiological point of view, the nervous system is an organ system “[...] specialized to conduct information in the form of impulses that controls, regulates, and coordinates all functions of the body” (Rothenberg & Chapman, 2000, p. 385). Beliefs about mind and its physiological basis have been a topic of debate for thousands of years, and are exemplified in the concept of Cartesian dualism (see Descartes, 1614/1985). Cartesian dualism has been greatly challenged by the neuroscientific observations of similar behavior changes of *Homo sapiens* who suffer from similar congenital brain abnormalities and or sustained intracranial and intercranial brain injuries (Purves, Augustine, Fitzpatrick, Hall, LaMantia, McNamara, & White, 2004).

Behavioral, etiological, and pathological investigations of deviation from the species-wide homogeneous morphology of molecular and cellular mechanisms as well as organs of the nervous system are compared with corresponding organismic behavioral change, and this comparison forms the basis for understanding the functions of brain regions (for a history of comparative anatomy, see Cole, 1944). While regions of the brain have been demonstrated to be central hubs of certain behavioral and regulatory functioning (see Purves et al, 2004 and Squire & Berg, 2008), neuroplasticity has been shown to allow the central (see Rohrer, Fasoli, Krebs, Hughes, Volpe, Frontera, & Hogan, 2002) and peripheral nervous system (see Behrman & Harkema, 2000) to adapt to injury in varying degrees. Connectionisms such as concentric theory (Greenfield, 1995) lend explanation for how regional hubs of neural functioning and neuroplasticity are related.

As shown above, there is overwhelming evidence that the anatomy of *Homo sapiens* functions organismically the same across the entire species, and we may suspect it has been this way for at least a median of 50,000 years. Neuropsychological studies have demonstrated that mental functions have direct correlations with various regions of the brain and other physiological functionalities, so it stands to reason that the scope of the internal, what some might call the mental experience of *Homo sapiens*, is predicated and determined by the neurobiological makeup of humans. Recent work has demonstrated that neuroimaging methods can be used to decode neurological processes of semantic categories into digital databases, and the results suggest that similar neurological processes of similar semantic categories are shared across different individuals (see Thirion, Duchesnay, Hubbard, Dubuois, Poline, Lebihan, & Dehaene, 2006; Miyawaki, Uchida, Yamashita, Sato, Morito, & Tanabe, 2008; Naselaris, Prenger,

Kay, Oliver, & Gallant, 2009; Pereira, Mitchell, & Botvinick, 2009; Pereira, Detre, & Botvinick, 2011). Neuroimaging of semantic categories is a very young neuroscientific topic, coming into existence within only the last century. But from the neuroimaging work already done, evidence yields strong support for the validity of architectonic modeling, because universal principles and or categories of experience and knowledge are just another way of defining shared semantic categories across individuals. I hypothesize that architectonic models, especially integral and recursive ones, rest at the heart of bridging and synthesizing phenomenology with neuroscience once and for all. In summary, this section offers some of the foundational scientific bases and rationale for this thesis project as well as the SHIM model, which is introduced in the next section.

The Spectrum of Human Imagination Model

In this section, I describe the conception of SHIM rendered as an integral architectonic in transition to become a recursive architectonic, for which this thesis project is a continuation. This introduction's usefulness depends on clarity about the meaning given to *qualia*. The term qualia is the plural of *quale*. By quale, I mean an attribute of experience produced by neurobiological perturbations. Generally I use the plural term instead of the singular because attributes of experience are almost always admixtures of more than one quale unit in the SHIM hypothesis. I did not borrow this term or define it from any scholar or school of thought – I searched through thesauri and dictionaries until I found a word that most closely resembled the type of category for which my model accounted, and augmented the definition for the SHIM context. I use Latin terms to distinguish the qualia categories in SHIM from assembled conceptions humans may form about them, and these are introduced throughout the following

discussion. Before continuing, I recommend readers glance at the figure and tables in this chapter in conjunction with the Appendix to orient themselves to the terms, content, and relations discussed here.

In brief, in building SHIM from the bottom up, I determined people have experiences through and of their organism, iterating neural connection networks that are activated viscerally throughout the human organism in the immediacy of an experience (monotypal qualia), and these experiences are coordinated into increasing admixtures of qualia with increasing magnitudes of complexity (nuotypes). I organized them by their attributes, and then put them into order by their natural relations that they shared in common (polytypes). The natural relations produced fractal holarchies (polytypes). These fractal holarchies shared common characteristics that were true at every tier in them, and I abstracted their common characteristics (omnitypes). I synthesized these common characteristics by their natural relations into even higher abstract principles and categories (metatypes). The highly abstract categories could be grouped together by higher abstractions (archetypes), and then finally by a category that bound the last three highest abstractions into a single, even higher abstraction (deitype). I then found from this overarching view, that this is all bound together by a single concept (quatype). The model, being an integral architectonic of all attributes of all experience shared throughout the species, hypothetically consists of all the components required to represent itself. However, it is the quatype that grants access to transitioning the model into a recursive architectonic. This bottom up view is only one view possible, the integration view; the next paragraphs describe the model from the top down view, the differentiation view.

From the top down view, SHIM is a tripartitic fractal holarchy, meaning that the units of the model are wholes and parts simultaneously (holarchy) and exhibit self-similarity to each other and to the whole (fractal). This universal character that permeates the model is what I call the quatype. The quatype is the unifying category that binds together the integral architectonic and gives it the capacity to become a recursive architectonic. The quatype has three fundamental characteristics that recurse: static, dynamic, and multinamic. This is why I characterize it as tripartitic. By static, I mean a maintaining state of some kind, and by dynamic I mean an activity of some kind. I have coined the term multinamic to describe the combined properties of static and dynamic.

Continuing description from the top down view, the quatype multinamism divides into seven categories of stratification from greater to lesser admixture of qualia. The contents of the quatype, in order from greatest to least mixed, are the deitype, archetypes, metatypes, omnitypes, polytypes, nuotypes, and monotypes. Four of the quatype categories, the higher orders of abstraction, compose the core model: the deitype, archetypes, metatypes, and omnitypes (see Table 1). Though I consider the omnitypes to be part of the core model, the omnitypes are the exact middle of the stratified categories of qualia abstraction (see Appendix), and it was the pivot point between my coordinations of the highest and lowest stratifications for which the quatype emerged as a conception. It was at this crucial time that my recursive operations on my model yielded the highest order principle, the quatype, and I realized that in order to advance my model, I needed to place the recursive operations I was performing on the model, as a performative part of the model itself.

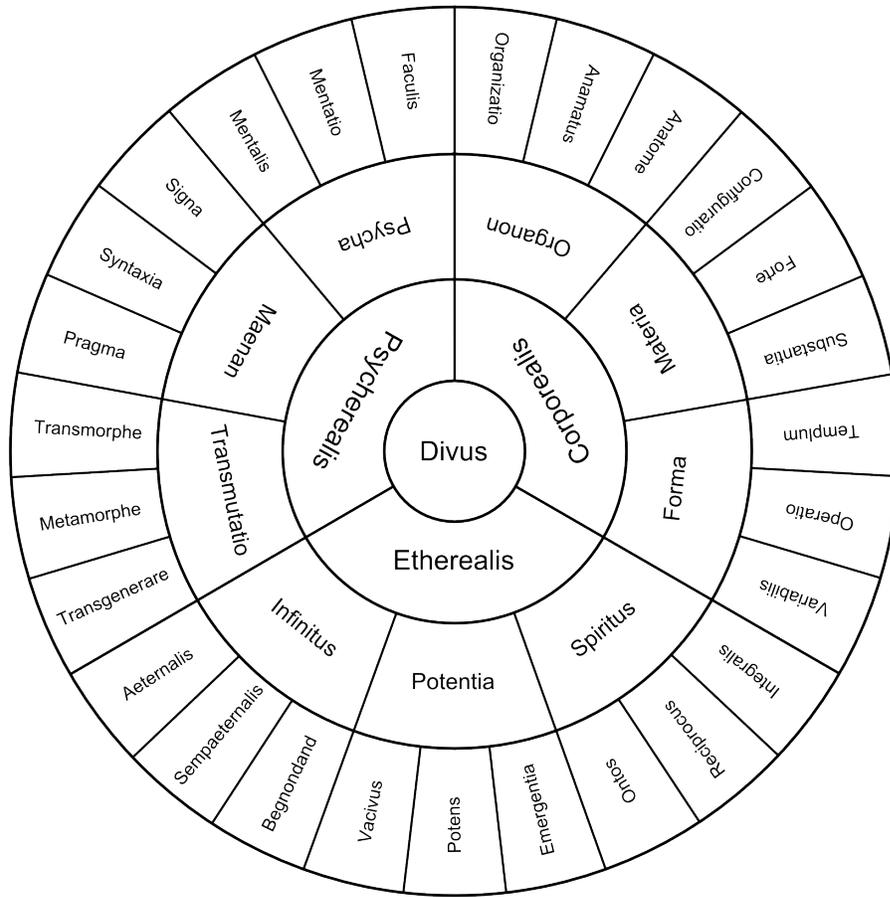


Figure 1. Spectrum of Human Imagination Model v84.8, Model Core. Copyright © 2012

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Table 1. The Spectrum of Human Imagination Model v84.8, Terms and Definitions.

Quatype: Universal abstraction, gradients as categories	Deitytype: Highest abstractions, multinamic admixture, static and dynamic are indistinct	Archetypes: Second highest abstractions, multinamic admixture, static and dynamic are indistinct	Metatypes: Third highest abstractions, multinamic admixture, static and dynamic are indistinct	Omnitypes: Fourth highest abstraction, multinamic admixture, static and dynamic are distinct, albeit very abstractly
Quatype: multinamism	<i>Divus</i> : qualia of deities and divine nature, self- generative, self- referential	<i>Etherealis</i> Archetype: qualia of intangibility	<i>Infinitus</i> Metatype: qualia of the unchanging, perpetual and beyond	<i>Aeternus</i> Omnitype: qualia of endless unchanging
				<i>Sempaeternus</i> Omnitype: qualia of endless perpetuation
				<i>Begondan</i> Omnitype: qualia of beyondance
			<i>Potentia</i> Metatype: qualia of emptiness, possibility, and manifestation	<i>Vivacus</i> Omnitype: qualia of emptiness, non-existence
				<i>Potens</i> Omnitype: qualia of possibility
				<i>Emergentia</i> Omnitype: qualia of manifestation
			<i>Spiritus</i> Metatype: qualia of being, reciprocity, and integration	<i>Ontos</i> Omnitype: qualia of being
				<i>Reciprocus</i> Omnitype: qualia of reciprocity
				<i>Integralis</i> Omnitype: qualia of integration
				<i>Variabilis</i> Omnitype: qualia of variable
	<i>Operatio</i> Omnitype: qualia of operation			
	<i>Templum</i> Omnitype: qualia of template			
	<i>Substantia</i> Omnitype: qualia of physical substance			
	<i>Forte</i> Omnitype: qualia of physical force			
	<i>Configuratio</i> Omnitype: qualia of physical configurations			
	<i>Anatome</i> Omnitype: qualia of anatomical units			
	<i>Anamatus</i> Omnitype: qualia of organic animation and interaction			
	<i>Organizatio</i> Omnitype: qualia of organization			
	<i>Forma</i> Metatype: qualia of variables, operations, templates			
	<i>Materia</i> Metatype: qualia of physical substances, forces, and configurations			
	<i>Organon</i> Metatype: qualia of organic anatomical units, animation and interactions, and organizations			

<i>Psycherealis</i> Archetype: qualia of intermixed intangibility and tangibility	<i>Psycha</i> Metatype: qualia of faculties, mentations and orientations	<i>Faculis</i> Omnitype: qualia of mental faculty
		<i>Mentatio</i> Omnitype: qualia of mental animation
<i>Maenan</i> Metatype: qualia of signification, syntax, pragmatism		<i>Mentalis</i> Omnitype: qualia of mental orientation
		<i>Maenan</i> Omnitype: qualia of meaning, signification
		<i>Syntaxia</i> Omnitype: qualia of syntactic relation
<i>Transmutatio</i> : qualia of facilitation, cause and effect, and temperance		<i>Pragma</i> Omnitype: qualia of pragmatism, context
		<i>Transmorphe</i> Omnitype: qualia of schemata
		<i>Metamorphe</i> Omnitype: qualia of transformational action
		<i>Transgenerare</i> Omnitype: qualia of temperance, result of transformational action

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The three remaining quatYPES compose the peripheral of the model: polytypes, nuotypes and monotypes. Figure 1 is a diagrammatic representation of the core model, followed by Table 1 with terms and definitions. In the context of keeping this chapter compact and directly in line with its purpose, I do not elaborate further on the definitions of the qualia categories at each stratification and or order of admixture. Nor do I provide examples of the presence of them from all times and places, or make correlations to neurophysiology. These are to be topics for future writing.

On the periphery of the core model are the fractal holarchies that extend from the core model. The fractal holarchies occur through the recursion of the omnitypes. These recursions of omnitypes I call polytypes. Polytypes are variations of the static, dynamic and multinamic character of the omnitypes from which they are iterations, but represent them at different magnitudes of qualia assembly.

Magnitudes of quatypal iteration are shared across polytypes. For example, a polytypal tier of 3 would be the same magnitude descending from the anatome omnitype as it is for the faculis omnitype, thus two qualia attributes of the same magnitude of qualia assembly. Their construction is shown in Table 2, where the polytypal tiers are shown as iterations of omnitypes, which stratify the different orders of magnitude of qualia. The total listing of the polytypal fractals of qualia magnitudes can be seen in the Appendix.

Table 2. The Stratification of Polytypal Recursive Magnitudes Represented in a Table.

		Metatype m_1			Metatype m_2		
		Static Omnitype	Dynamic Omnitype	Multinamic Omnitype	Static Omnitype	Dynamic Omnitype	Multinamic Omnitype
Polytypal Tier n_1	Polytypal Tier 1 Static: A as part of a greater whole	Polytypal Tier 1 Dynamic: B as the interaction between variations of A	Polytypal Tier 1 Multinamic: C as the whole of A's interacting through B's	Polytypal Tier 1 Static: P as part of a greater whole	Polytypal Tier 1 Dynamic: Q as the interaction between variations of P	Polytypal Tier 1 Multinamic: R as the whole of Q's interacting through P's	
Polytypal Tier n_2	Polytypal Tier 2 Static: C as a part of a greater whole	Polytypal Tier 2 Dynamic: D as the interaction between variations of C	Polytypal Tier 2 Multinamic: E as the whole of C's interacting through D's	Polytypal Tier 2 Static: R as a part of a greater whole	Polytypal Tier 2 Dynamic: S as the interaction between variations of R	Polytypal Tier 2 Multinamic: T as the whole of R's interacting through S's	

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Each polytype has very specific types of content that meet the non-arbitrary positioning in the particular tier/magnitude within the fractal holarchies of polytypes. I call these assemblies nuotypes. Nuotypal static, dynamic, and multinamic assemblies are within polytypes, and are quatypal iterations that go into, across, and perform loops through polytypes. In Table 2, nuotypes are the content of the variables. Nuotypes are aggregates of qualia into groupings. In SHIM, these groupings are the units of conceptual interfacing such as classes, symbols, and conceptual representations which humans depend on to interact with themselves and their environment, and through which even SHIM is represented.

Nuotypes are groupings composed of monotypal qualia units. Monotypes are static, dynamic, and multinamic. I hypothesize monotypal qualia assembly as the iterating neural connection networks that are activated viscerally throughout the human organism in the immediacy of an experience.

The reason why SHIM is important to this thesis project is because SHIM hypothetically provides a framework for describing all the attributes for which knowledge and experience are assembled, and SHIM is in transition to describe the recursive process of understanding and mapping those assemblies. The next step in the development of the model is to create a method to represent this process computationally, by its highest order principles of the quatype, and for this I propose the FPC.

The Relation of SHIM and FPC

To introduce the connection between SHIM and this current effort, I begin by defining more terms. I give definitions of each individual word in the term *fractal phase calculus*, and then give a definition of the FPC as a whole. By phase, I mean a temporary

form of some kind. By calculus, I mean that phases are constructed by certain groups of things, and the things are put together to form those phases by following certain rules. There are different kinds of phases with different rules with different complexities. By fractal, I mean a recursive process and or architecture where the parts exhibit similar characteristics as the whole. Fractal phase calculus, then, is for describing universal rules that all things, their relations, and phases devised from them, express. I am proposing quatype computational notation.

This is essentially an attempt to create a model to universally represent every architecture and process of every phase calculi that has been conceived, are being conceived, and could ever be conceived by our homogenous anatomy. In order to do this, the FPC must satisfy Gödel's completeness and incompleteness theorems so that it is true for all conceivable complete and incomplete calculi, in such a way that it would *define* itself *improving* itself as part of that completeness. I hypothesize that the evolution and or development of humanity amidst all its diversity, for whatever reason, is now rooted in universally shared anatomical form, and the recursivity that is intrinsically embedded within and drives the evolution and or development of individuals, groups and the entire species, can be understood and utilized to help catalyze the positive evolving of humanity as a whole.

CHAPTER III

LITERATURE REVIEW

The purpose of this chapter is to put my thesis project in its context of the greater scope of related literature and describe how I expect my research to contribute to new knowledge. My research aim is to develop a fractal phase calculus (FPC) that can be used as the computational basis for recursive architectonic software. I give brief historical accounts of key innovations in mathematics and logic that led up to and continued through the development of computability theory and behavioral complexity science, and then describe how my thesis project relates to these topics.

Computability Theory

In the 17th century, Leibniz (1684/1969) devised calculus independently but simultaneously as Newton (Bardi, 2006). While both of their variations of calculus and their impact on human knowledge are important, for this literature review, the Leibniz contribution is of particular relevance. Leibniz devised calculus with two important things in mind. First, he believed it was possible to create a universal language for the symbolic representation of anything, a *characteristica universalis*. Second, he invented the concept of a universally-applicable calculatory framework for the purpose of expressing this, which he called a calculus ratiocinator, and designed a calculation machine based on that framework to perform simple mathematical operations. His

expressed hope was that someday such a machine could perform like-operations with a *characteristica universalis*. His reasoning was that it would take several years with many people collaborating in order to design such a thing, and that it was not something he could complete by himself. However, this never happened. Whatever *characteristica universalis* Leibniz did prototype, seems to be missing (Dawson, 2005, p. 189). Though there were people who came before and influenced Leibniz or computer science through other channels, Leibniz's work is important because the calculus ratiocinator and *characteristica universalis* would lay the foundation for all of computer science through the innovations that were made possible by his contributions. This lineage is described below.

Leibniz's inventive design for the calculus ratiocinator led to others contributing further innovations. For example, Boole used Leibniz notation in *The Laws of Thought* (1853), where he outlined binary logic. Frege published *Begriffsschrift* (1879) to create a *lingua characteristica* from the calculus ratiocinator, an answer to Leibniz's *characteristica universalis*, which established formulaic logic (Korte, 2010). Boole's calculus of logic is referenced several times in Peirce's *Studies in Logic* (1883), and Peirce's work laid the grounds for semiotics. Binary logic, formulaic logic, and semiotics all depended on calculus.

It is widely known that Hilbert was influenced by and supported Kantian intuitionism, and furthermore supported Cantor's contributions (Hilbert, 1926). From 1900 on, Hilbert presented several problems he considered necessary to be resolved in the field of mathematics (Newson, 1902). One of these problems was called the decision problem. The decision problem was basically this: can a set of axioms be used to decide

if a statement is universally true or not? Hilbert gave a presentation about these problems, including the decision problem, in Vienna in 1929. Gödel was present, and responded to this problem (Ferferman, 1994). From 1929 to 1931, Gödel devised and published his completeness and incompleteness theorems (Heijenoort, 2002). The simple version of these theorems is that they showed that a formal system cannot be complete and consistent at the same time.

In the mid 1930's, Tarski had taken Gödel's theorems and applied them to the relationship between semantics and mathematics, and published on the undefinability theorem, which stated that arithmetical truth cannot be defined in arithmetic (Tarski, 1936/1983). This is important to mention for the proceeding culminating description of my effort in relation to the literature below, because this seems to have been the first explicit articulation that Gödel's theorems are not just mathematical, but are equally applicable to semiotics.

Meanwhile, Church had been studying the foundational basis for mathematics, and in conjunction with Gödel's theorems, devised lambda calculus that consisted of types, functions, and recursivity (Church, 1936). Turing had been working on calculability problems, and in conjunction with Gödel's theorems, independently drew the same conclusion as Church in relation to his hypothesis for intelligence machines (Turing, 1937). Soon after, Turing made the connection between his and Church's work (Turing, 1937), and began his PhD under Church at Princeton. In 1938, Turing's PhD dissertation, supervised by Church, was entitled System of Logic Based on Ordinals (Turing, 1938/2012). In it, he explored the infinite recursivity of Gödel's theorems and described a means of mathematically tracking the recursivity of logics. This was a very

important contribution, because it was perhaps the first effort to track the transformation of logics over time and integrate them together. And in 1940, Church developed untyped lambda calculus (Church, 1940); for the first time, humans had a system for universal calculability.

McColloch and Pitts (1943) were influenced by the contributions towards the advancement in logic of Carnap (1938), Hilbert and Ackermann (1927/2008), as well as Russell and Whitehead (1925). They wrote a paper entitled *A Logical Calculus of the Ideas Immanent in Nervous Activity* (1943), which described a computational neural network theory. What these three major contributions have in common is their dealing with the fundamental basis for logic and mathematics. It also seems important to mention that Carnap's *The Logical Syntax of Language*, cited by McColloch and Pitts, makes many direct references to Gödel's work in his book. Thus, we may suspect that McColloch and Pitts were well aware of Gödel's theorems even though their 1943 paper does not mention Gödel directly. This is important to mention because Neumann's work on blueprinting the electronic discrete variable computer (EDVC), from my historical analysis, seems to have been the first modern computer to bring many contributions together and set a major benchmark in computational machines in three ways: (a) making use of Boolean logic by moving away from decimal and into binary programming by Boole's binary logic (Neumann, 1945, p. 6); (b) configuring the computational model as a logical calculus by McColloch and Pitts' major contribution to neural network theory (p. 5); and, (c) applying Turing's recursivity (p. 1).

In 1946, Gödel attended a presentation given by Tarski at Princeton, and was cited as agreeing with the notion that Turing's recursivity was an epistemological notion

that applied across formalisms (Hodges, 2012, p. 112). Gödel had also become aware of Leibniz's concept of *characteristica universalis*, collected every known manuscript Leibniz had written on the subject, and thought favorably of that body of work. In fact, Gödel, noticing that Leibniz had made references to the *characteristica universalis* in manuscripts that were missing, was deeply troubled by their apparent loss; and Gödel became convinced that the missing manuscripts were purposefully made inaccessible (Dawson, 2005).

By the 1940's, multiple programmable computers had been developed. Until Backus spearheaded the development of the formula translation language (FORTRAN) (Backus, Beeber, Best, Goldberg, Herrick, & Hughes, 1956) computer languages were low level with code tied directly to the circuitry operations. FORTRAN was a computational language that allowed both programming and compiling at a much higher abstraction than what had previously existed. Though the original FORTRAN manual does not cite references, and though I was not able to locate Backus' master's thesis in mathematics, it is clear upon inspection of the manual that FORTRAN inherited and applied the discoveries from pioneers mentioned above. FORTRAN introduced practical application for high level programming through the implementation of a compiler. This is important, because compilers yielded the capacity to convert high-level languages to low level languages and vice versa. In other words, a high level coded language could be converted into machine code, and then converted into another high-level language. Further, it is important to note that FORTRAN made use of multiple data types, valuable because it allowed for different semantics to be expressed in the same language.

In 1958, McCarthy, Brayton, Edwards, Fox, Hodes, Luckham et al. (1960) developed the LISP processor (LISP). LISP was another high level programming language. McCarthy explicitly stated that LISP was based on Church and Turing's work, and that he was also aware of FORTRAN. His diagrammatic representation of list structures (McCarthy, 1960, p. 23) bears a strong resemblance to Frege's (1879) logic notation. The clear articulation of list structures, better known as data structures in today's language, is an important computational notion. McCarthy cites Newell and Shaw as the source of his concept of list structures, where the same Frege-like diagrammatic representations can be found (Newell & Shaw, 1957). These developments clearly trace a common thread from Leibniz to our time, to which I return to below. While the earlier generation of codes, FORTRAN and LISP, are still in use today, it led to subsequent generations, where today we have a rather broad genealogy of computer languages (Boutin & Hailpern, 2002).

The relevance of this historical trajectory in computational languages is more apparent when accompanied by a discussion of hardware and software interdependence. It is quite intentional that hardware circuitry and software language are at base both binary in the operation of computational machines via Neumann as discussed above. Circuits are binary because circuits either do or do not send a signal. Boolean logic is used for lower level programming because it perfectly matches the binary circuitry activity in the hardware. Lower level Boolean mathematical logics are converted into high-level languages because people usually do not think and communicate in binary. High-level languages allow people to write code in more human-like ways, e.g., two widely used computational languages, C++ and Objective C. C++ rests on the basis of

variables and operations that are divided into various sub-types. Variables come in primitive data types such as boolean, integers, and characters, while operations come in primitives such as arithmetic and comparison (Stroustrup, 1997, p. 24). Objective C likens programmed objects to real world objects, whereas these objects are described as having states and behaviors, which is equivalent to data and operations on the data. Objective C has different means of implementing code, but shares in common these same data types (Apple Computer, Inc., 2002). In both cases, the data types are more like how we think; for example, integers are the symbols we use to represent mathematics, and characters are the symbols we use to represent our spoken language. But what is not so obvious is the relationship between the power of calculus and the human behavior emulated in machines. After all, Alan Turing's reference to computational machines as intelligence machines from the 1930's makes it clear that even then, behaviors of these computational machines were recognized as emulations of human behaviors. Tarski, as mentioned above, showed how Gödel's theorems applied not just with mathematics and logic, but with semantics (and semiotics) as well. So to answer this question, we have to look at behavioral complexity.

Behavioral Complexity

Piaget contributed much to understanding human behavior, but three of his key innovations have central importance here. First, he devised cognitive stage theory. In 1928, Piaget published a book entitled *Judgment and Reasoning in the Child*, using logic architecture to describe Piaget's observation that children had an "absence of logical hierarchy or of synthesis between different elements of the same conception" (Piaget 1928, p. 156). Though at first this distinction between reasoning in children and adults

was rather rudimentary, dividing reasoning into the two categories of child and adult, over the next several decades Piaget and Inhelder conceived of four stages of cognitive development: sensorimotor, preoperational, concrete, and formal. This was important because it was the first time that humans had articulated a concise hierarchical architecture for cognitive development by means of logic architecture.

Second, Piaget began to use the concept of equilibrium to indicate interplay between static and dynamic cognition. Such concepts underpinned *The Growth of Logical Thinking* (Inhelder & Piaget, 1958). Piaget and Inhelder (1973) later outlined a more complete picture of equilibrium with static and dynamic. They described two kinds of cognitive behavior: figurative and operative. Figurative behavior was static and represented things, and operative behavior was dynamic and operated on things. Subsequently, this line of work was refined to present the development of thought as equilibration of cognitive structures (Piaget, 1977), where Piaget linked the cognitive static and dynamic characteristics with biological statical and dynamical characteristics. Here, the important point is that Piaget used static and dynamic to define kinds of cognitive behavior.

Third, with Inhelder, Piaget devised a model of transition between developmental structures or stages. The transition consisted of four steps: disequilibrium, accommodation, assimilation and equilibrium; at equilibrium, a new stage would be attained (Piaget, 1973, p. 36). Though it is true that syllogistic logic can be traced as far back as Aristotle (Hope, 1952), and its conceptual potency was revived by Kant (1781/2004) and present in all major literature on logic since then, Piaget's

implementation of the synthesis between disparateness is of great importance because he identified this pattern's identical recurrence between each of the cognitive stages.

Piaget's genetic epistemology is commonly recognized as founding the field of development theory, rooting it in empiricism. One development theory of great importance here that maintained such empiricism is the model of hierarchical complexity (MHC; discussed below). Commons essentially picked up where Piaget had left off (Commons, 2008). The MHC was a reconstruction of Piaget's stage architecture, providing a mathematical framework rather than Piaget's logical framework. This process corrected mistakes Piaget had made (Commons, Richards & Armon, 1984; Commons & Richards, 2002) bringing more precision to the measurement of stages of behaviors.

In 1982, Commons, Richards, and Kuhn showed that a stage of cognitive behavior existed after Piaget's formal stage (Commons, Richards, & Kuhn, 1982). In 1984, Commons, Richards and Armon published the general stage model (Commons, Richards, & Armon, 1984), which would later come to be called the model of hierarchical complexity (Commons, et al, 1998). By using mathematics, they showed that each order of complexity was produced by the behavioral coordination of the previous order's behaviors, and that many of Piaget's sub-stages were not sub-stages, but stages themselves. This is important because it allowed a content free stage model, free from any particular kind of content or behavior (e.g., Piaget's schema) while retaining the general patterns that behaviors can take, stratified as a hierarchy.

Commons and Richards (2002) showed that the transition between stages included a transition step between Piaget and Inhelder's assimilation and equilibrium

steps, which Commons and Richards called *smash*. They divided up the transition steps into deconstructive (steps 0-2) and constructive dialectics (steps 3-4), and described smash as consisting of three sub-steps that described the challenge of coordinating a synthesis from the components (Commons & Richards, 2002, p. 163). This is important because it was a benchmark in the advancement of human understanding of how transitions between orders of complexity happen.

Ross (2007; 2008a) showed that the transition dynamics and orders of complexity followed a universal, fractal pattern. She demonstrated that just as the MHC held for machines, small and large organisms and social groupings, so too did the universal transition dynamics pattern. She also introduced the notion of sub-tasks and sub-sub-tasks. While Commons (Commons, Gane-McCalla, Barker, & Li, in press) recently discusses sub-tasks and describes them in a combinatoric context, Ross's description of sub-tasks stipulates a fractal context. These novel advancements are important because she identified not only that the patterns existing between orders of complexity were fractal, but she also identified measurable fractals of sub-task dynamics and hypothetically unlimited levels of sub-sub-task actions possible in more complex entities' transitional states. She has proposed (in press) the need to integrate fractal transition theory into a nonlinear model of hierarchical complexity and explained how the fractal version corrects conceptual and measurement problems in Commons & Richards' (2002) smash transition steps.

While fractals are increasingly recognized in such domains as physics, biology, and financial markets, this appears to have been the first time that fractals had been studied in development theory. Though Mandelbrot and Piaget had written *Logic of*

Equilibrium together in 1956, Mandelbrot appears to not have fully formed his concept of fractals until the late 1970's (Mandelbrot, 1977), nor does it appear Piaget fully formed his concept of the equilibration of cognitive structures until the same time period (Piaget, 1977), neither author making any reference to the other in their work. Thus, it seems the connection was not made between fractals and transitions and the fractal nature of the model of hierarchical complexity itself until Ross.

Other Areas of Literature Search

I have reviewed a considerable number of articles that span both computability theory and behavioral complexity theory in the recent literature. However I have found none that continue the lineage of major innovation beyond the significant foundations laid by the classic to contemporary works discussed above. Other innovations of thought and application are indirectly related to my objectives in this thesis.

I reviewed the field of propositional and fractional calculus, for example in Goldrei (2005), Edgar (2008), and Herrmann (2011). However, these samples and other calculi in general are for specialized purposes and therefore lack the universal calculability that only untyped lambda calculus and Turing computability are able to contextualize and describe.

My literature search and review of the philosophy of mathematics was extensive and included but was not limited to, fictionalism (Bostock, 2009), structuralism (Shapiro, 2000), and paraconsistent mathematics (Colyvan, 2012). In my view, these philosophies each contributed useful perspectives on the topic, but taken alone express domain disparateness that does not support my work. It is my opinion that philosophy of mathematics is in desperate need of empirical groundings in neuroscience (Buttersworth,

2002), and with it the disparities of philosophy of mathematics synthesized in a transdisciplinary manner. Further, it should be said that development theory was not referenced in relation to philosophy of mathematics in any of the literature I reviewed. Such a transdisciplinary synthesis of the literature into a unified framework is of much interest to me, but is not directly pertinent to my thesis.

Foundations of mathematics literature was reviewed, such as Bittinger and Penna (2004), Mustoe and Barry (1998), and Wilder (2012). However, like philosophy of mathematics, transdisciplinary considerations for neuroscience and development theory appeared to me to be completely missing from the foundations of mathematics literature. Further, fractal mathematics seemed to be treated as an appendix to the greater scope of mathematical knowledge, and thus treated as a separate, special branch of mathematics rather than giving it special attention as the inherent mathematical properties of the universe that it seems to represent (Mandelbrot, 1977). The literature search required additional items such as Falconer (2003) to gain an adequate overview of the measure for which fractal geometry has grown as a field. Due to these unfortunate circumstances of this stage of knowledge development, the literature was not helpful to my thesis objective.

In addition, I extensively reviewed contemporary literature on human-like behavior and knowledge representation being programmed into and emulated by computational machines. It was not so much that useful efforts are not being made in a diversity of areas to create machines that emulate human-like behavior, such as discussed in Bregnant and Aberšnek (2011), Cristina, Beatrice and Florentina (2008), and Vassiliadis (2009). But rather, my observation was that the extent to which the artificial

general intelligence (AGI) community references development theory stops at Piaget and Vygotsky. I was thus not surprised that Adams, Arel, Bach, Coop, Furlan, Goertzel, et al.'s (2012) survey of the AGI landscape stressed the importance of cognitive development to the future of AGI, but mentioned not one developmental theorist beyond Piaget and Vygotsky. It would appear that the AGI community is over 30 years behind on the current developmental, cognitive, and behavioral complexity literature. And as for the ontological knowledge representation, I found nothing that even remotely resembled an integral or recursive architectonic, and therefore no references can be given.

Summary

I suggest Tarski was right when he proposed that Gödel's theorems are not constrained to mathematics only. I propose this because my analysis indicates Gödel's theorems are synonymous with the orders and transitions in the model of hierarchical complexity, and therefore recursively play out at and between every order of complexity: completeness as the orders and incompleteness as the transitions. I propose that Ross' fractal transitions describe untyped lambda calculus in action between the alternation of completeness and incompleteness. I conjecture that Gödel's theorems and Church-Turing computability should no longer be described just as properties of mathematics and logic, but of human behavior in general.

Since computational machines were designed with the purpose of emulating human intelligence, it follows that in order to advance computability theory, a major next step in the design of computational machines would be to use our understanding of the complexity of behavior as the organizational basis for the operation and data structuring in those computational machines. In the next chapters, my method for doing so is

described, a computational description outlined, an example of application demonstrated, and implications discussed.

CHAPTER IV

METHODOLOGY

The purpose of this chapter on methodology is to convey the approach I used to develop fractal phase calculus (FPC) for its expression in computer-usable language. My goal was to bridge extremely high-level abstractions into a language computers can understand. Thus, I matched FPC properties to mathematical languages and their functions that computers can understand to conceive and produce the FPC notation and then deploy it in developing the resulting systems of axioms and theorems presented in Chapter V.

In this chapter, I reference qualia and complexity by drawing from the spectrum of human imagination model (SHIM), and the model of phase complexity (MPC) (Barker, in preparation). The spectrum of human imagination model is a content-descriptive integral architectonic model that accounts for, describes, and organizes knowledge about universal qualia abstractions and qualia categories across all domains of knowledge and experience of the human species. The model of phase complexity is an integral architectonic content-free model that accounts for, describes, and measures the complexity of universal orders of action and phase transitions of action across all domains. In that way, MPC helps organize knowledge and is applicable to all domains of knowledge and experience of the human species. These two integral architectonic

models became the basis for recursive architectonic synthesis. Since I persistently make use of MPC for the remainder of this thesis, a brief description of MPC is warranted.

MPC builds on and expands the MHC notions of order and transition in several important and novel ways, which I describe here by comparison. First, whereas MHC describes orders of complexity of machine and animal/human behavior, MPC describes orders of phase complexity that are both scale-free and universal to entities of all kinds. By scale-free, I mean to convey properties that exist at all levels of building blocks of nature. Second, whereas MHC describes horizontal and vertical complexity, MPC introduces diagonal complexity, which yields explanatory power for describing both nonlinearity of action, and the process of the construction of any entity at any scale. Third, MHC uses combinatorics and abstract algebra to describe vertical and horizontal complexity, but never incorporated either transitions or the fractal dynamics of transition between orders into its formal theory as Ross advocated for (Ross, in press). MPC inherits the combinatorics and abstract algebra of MHC, but completes the motion of incorporating the fractal dynamics and uses calculi to open the door for fine-grained analysis of sub-tasks and sub-subtasks. In summary, while MHC describes linear, single scale descriptions of behavior of machines and organisms, MPC describes multi-linear and scale-free descriptions of phase complexity and entity construction across all entities.

Due to the non-linear process of devising FPC notation, it is impractical to give a completely linear account as if it were a sequential process. Instead, I structure this chapter by discussing the primary components of FPC, the issues relevant to them, and how I resolved those issues. My notation in progress consists of the following four components: multinamics, recursivity, n-dimensions, and indexing, which I devised by

the orders of qualia abstraction (see Chapter II), and phase complexity. I discuss these four components below.

Multinamic

The first representation I needed to develop was for multinamics. The pure quatypal representation, as I expressed the quatypal properties, was multinamic "⊙" (a synthesis of both static and dynamic), static "●", and dynamic "⊖". This type of notation is needed because the quatypal qualia properties had to be distinguished from meta-semiotic, meta-logic, and meta-mathematical notation. And this is because the quatypal qualia properties were more abstract than properties describable with these other types of notation and thus were present within them all. Thus, these properties' notations could not be representable by meta-semiotic, meta-logic, or meta-mathematical notation systems without downwardly assimilating them into lower orders of phase complexity. Through the lens of MHC, those other meta-notation systems tend to score at only the metasystematic order of coordinating systems. By contrast, FPC needs to coordinate information from a much higher order of abstraction and thus a much greater order of complexity. This is because FPC was posed to describe the underpinning qualia of all orders and transitions between them, scaling all the way up and down the hierarchy, across and for all domains in the recursive architectonic task. In summary, then, the quatypal representation I developed had to build in universality.

The recurring but unrecognized practice of downward assimilation of the universality intrinsic to metatheoretical semblances capable of describing other representational systems has—in conjunction with a complete absence of an integral architectonic—in my view, led to people over-exaggerating their representational

systems. Examples are logicism that states that logic underpinned mathematics and semiotics, or platonism that states that mathematics underpinned semiotics and logic, or nominalism that states that semiotics underpinned logic and mathematics. This was a problem that I discovered during my literature review, which should have been resolved by Gödel's and Tarski's paradigmatic modelings, had philosophers of said semblances then and now understood the full implications of Gödel's theorems and the work of Church and Turing that followed. I call this issue, for lack of an existing term, the *meta-superiority problem*.

Continuing forward, multinamic was found to not be the same as binary. Binary is a phase complexity at order 0 in MPC, describing the interaction and combinatorics of presences (1's) and absences (0's). Multinamic is inherent in both absence and presence, and in the phase transition of their relation. With MPC, for every vertical, horizontal, and diagonal phase complexity, at and from one scale to another, the phase transitions are composed of temporary equilibria (static) engaging in interactions (dynamic) to create higher orders of phase complexity across scales as a fractal (multinamic which takes on static and dynamic properties for the next higher order in this iterative process). In terms of SHIM, presence is fundamentally *ontos spiritus etherealis* qualia by definition of *ontos* as pure being, and absence is fundamentally *vacivus potentia etherealis* qualia by definition of *vacivus* as pure emptiness. SHIM describes both *spiritus* and *potentia* metatypes as each having their own respective consistence of three omnitypes of lesser abstraction. So how could it be that each metatype could have its own static, dynamic, and multinamic variations across polytypal fractals, if it were not true that the quatype does not express itself through them all? And how could it be that binary could transition

through phase complexity diagonally if multinamic wasn't inherent in transitions to begin with? Additionally, the binary operands on integrated circuits follow neural network theory and its use of *not*, *or*, *and*, and *while*, albeit lacking the clear state transitional organization that Piaget, Commons, and Ross alluded to.

The Occam's razor is that multinamic, that is to say quatype, is more abstract than binary and transitions. The reason why this was important, was because it strongly suggests that binary is a lesser abstraction than quatype, and therefore gives insight into how the recursive architectonic can be converted in a computational language and its binary operands.

Recursivity

In SHIM, whereas multinamic affinity was quatype, I found that recursivity affinity was indeed deitypal, the next descending qualia abstraction from quatype. Preliminary observations predict that these properties are true across all orders of phase complexity and phase transitions across all scales of the building blocks of human representation of phenomena. In other words, SHIM and MPC hypotheses are matching observations. A description is warranted because these two affinities, that is, quatypal multinamic and deitypal recursivity, explain and predict a pattern that would be observed across any architectural and processual models that describe universal properties. This is because due to the positioning in the SHIM ontological representation of qualia abstraction, these affinities would necessarily have to be present in all human behaviors and thus their patterns.

As a phenomenological hypothesis of human qualia, from the top down, deitype as a qualia describes self-reference. At its high abstraction, there is nothing to reference

but its own properties and what it inherits, its inheritance being only quatypal properties. When only quatype and deitype are allowed into the conception, the multinamic is indistinguishable from itself and may only be made distinct by means of self-reference. Thus, it should be no wonder why I chose to call this self-referential qualia “deitype,” since the historical representation of this qualia has been an attribution given to divinity and divine nature, *divus*, a qualia that goes beyond even infinity.

I speak neither for nor against the existence or non-existence of divinity here, only to say that in any cosmogonic representation of a divine being creating the universe, such representation will be found to exhibit some allusion to a divine being existing alone, and through self-reference to itself and nothing else, bringing about reality. Even in cosmogonic representations lacking a divine being instigating reality as found in Jainism, reality is given self-referential and self-regenerative character as the universe continually goes through a cycle of renewal. Why is this important to mention in this methodology section, one may ask? Because in developing the ontological architectures and processes of a recursive architectonic, the fractal phase calculus notation and the related axioms must be laid out by means of the sequence of inheritance to which universal properties correspond. From the top down perspective, the deitypal qualia of self-reference inherits quatype only, and is what I propose generates the fractal n-dimensions, which in their most concrete monotypal form, inherit the deitypal qualia property within them (n-dimension is introduced in the next section).

Any multinamism of human experience with any arrangement of static and dynamic properties necessarily exhibits deitypal properties, and therefore any unit of quatypal assembly can plug into any other unit of quatypal assembly, not just across

SHIM, but across any order of phase complexity and phase transition of the MPC. This qualia has a historical pattern of being associative to cosmogonic notions, but here I propose this deitypal qualia is an innate primordial property of human experience in general, and a quale that is the genesis of input to interior process to output for any given entity. As can be true with qualia in general, deitypal qualia is often recognized in some semblance, often downwardly assimilated and represented through lesser orders of complexity. However, in its most pure character, deitypal qualia is the universal property of which any entity (multinamic) can become the center of everything in part or whole of human experience. My observation indicates that in the 15th order of hierarchical complexity called performative-recursive or metacross-paradigmatic (Ross, 2008b; Ross, Commons, Li, Stålné, Barker, in press; Ross & Barker, 2013), this qualia serves as the guiding principle for the parts of a whole to continually inform the whole, and the whole regenerating, reorienting, and restabilizing its parts by means of the parts being in equal standings with the whole.

I chose the sign “ α ” to represent this recursive multi-linearity. It represents the universal channel of transference for which all qualia and complexities traverse. Any recursive multi-linearity can exist in relation with any other entity (multinamism) from any qualia assembly scale with any phase order and phase transition limited only by the natural constraints of the things to which a channel belongs. The epitomous worldwide symbol of this phenomena is commonly known as the ouroboros, and in agreement with Ross (personal communication, 2012) though here I express it in its SHIM context, and though in Table 3 and the Appendix I do account for mouth, body and tail, in its purest form no distinction between mouth, tail, and body can be made, as the qualia is more

abstract than the three archetypes for which conceptual representation of tangibility and intangibility requires.

N-dimension

Traditionally, the concept of n-dimension arose in mathematics to designate positions in multi-dimensional space. I use the term here in a related but more specific way, which this discussion introduces. N-dimensions help describe observations of the natural consequence of the self-referential property of deity within multinamism. The only thing that self-reference can do when all lower qualia abstractions are excluded, is reflect on its self, and in doing so, the n-dimensions are rooted. Humans experience their body's interior perturbations from external activity. In other words, humans' bodies are constrained to the organismic architectures and processes, and humans know the universe only through their bodies' constraints. So, to situate this phenomenological hypothesis of human experience of qualia into something more directly accessible, I mean specifically that our interior coordinations throughout our nervous system are constrained by the organism that we are. Here, n-dimensions are used to describe paths of nesting of fractal phase complexity across scales of qualia assembly.

SHIM with MPC in recursive architectonic semblance, together, describe three types of dimensions: horizontal, vertical, and diagonal. Horizontal and vertical are inherited from Commons et al.'s contributions to behavioral development theory, but diagonal complexity a term I have coined myself. Horizontal dimensions describe side-by-side entities, often with actions or entities of the same scale of phase complexity. Vertical dimensions describe the phase transitions and algorithmic results that occur in the interactions between entities at lesser, equal, or greater scales and orders. My notion

of diagonal dimensions are here described as downward and upward assimilation of the products of orders and transitions to other orders of action and their transitions – the means of tracking the behaviors over time. Together, these three kinds of complexity are coordinated with the qualia of the multiscalarity of the building blocks of entities (static) and actions (dynamic) captured by SHIM.

To account for these horizontal, vertical, and diagonal dimensions, I describe them as n-dimensions. N-dimensions occur in these three ways, so that I make the distinction as h-dimensions, v-dimensions, and d-dimensions, respective to the first letter of the type of dimension being referenced. This allowed me to account for the direction of the dimensions, organize the multiplicity of fractal nesting, and clearly articulate the relationship between them. Commons (M. Commons, personal communication, 2012) recommended my using linear algebra, and this was sufficient for my needs.

Indexing

I found that there is no such thing as a best notation for the quatyplal qualia. The quatype properties are inherent in all notation and in our behaviors for designating them, so that a best signification and syntax schema were relative to the function of a given use, which is to say that reality is the best example of itself. But given that the goal of this work was to bridge the abstraction into language computers can understand, the issue of bridging the abstraction was the problem of indexing.

In the SHIM hypothesis, organismic anatomical units and their interrelated animations (*organon*) perturb gestalting psychic architectures and processes (*psychia*) which are frameworks for which meaning making (*maenan*) are assembled into schemas (*transmutatio*). These schemas (*transmorphe transmutatio*) interact with other schemas

(*metamorphe transmutatio*). In MPC terms, the way entities interact is defined by their phase order complexity and phase transitions. In the recursive architectonic synthesis between SHIM and MPC, the entities are schemas, and the way that the schemas interact is defined by the order of their phase complexity with which they are coordinated by the entity of which they are a member; in this case, people are the coordinators.

Sometimes a schema is the basis for representing another schema. For example, the recursive architectonic is a schema. But the recursive architectonic is a schema proposed to measure the qualia and complexity of all schemas, and therefore it has to index itself. The realization that there was no such thing as a best notation posed a problem, because in order to mimic recursive architectonic behaviors, a computer would have to have a notation.

To solve this problem, I represented qualia abstractions as a fractal nesting of matrices, where qualia abstraction matrices were composed of qualia abstraction matrices, from quatype to monotype, each unit of the matrices being a multinamic unit. The matrices were given n-dimensional coordinates, where I always tried to keep 0 as close to the central median of human experience where our skin meets the air. Though I roughly describe this visually in the Appendix, it was impractical to give further figural representation because the n-dimensions exceed what can be represented in spatial dimensions, and the folds in the mappings of recursing multi-linear qualia assembly would look perhaps similar to a figural representation of a Calabi-Yao manifold (see Im, 2008). The visceral neural network threading through and of organ systems as the whole person is the best representation of itself.

I then used the fractal coordinate system as a unit of itself as a schemata. This allowed me to use the fractal coordinate system to account for itself, account for other schema that may represent the recursive architectonic through those schema, and any schema to account for any other schema, while respectively preserving the schema as it is presented. Any indexing (as a schema) could then be used to represent the recursive architectonic, while treating the recursive architectonic equally to the alter schema, paradoxically through the recursive architectonic.

Three hypotheses arose. First, this would allow infinite recursivity of all the parts to be capable of participating in interactions with any other, albeit by their interior constraints (or not!) while preserving earlier states from previous recursive iterations. This would be like memory. Second, the indexing could be converted into any computational language that obeys untyped lambda calculus, and so it is more a question of code efficiency and machine processing power for which I will make no guesses until I experiment with my results, post-thesis, in computational environments. Third, insofar as this hypothetical model works, any schema can be the center, mimicking the real world fact that each person has his/her own brain/mind distinct from other people with unpredictable starting points for any endeavor. The hypothesis is that the novel difference here from what already exists, is that the recursive architectonic, through this FPC, not only lends to explaining how universal calculability across domains happens, but gives a basis for and organizes the qualia and complexity of input into a schema that can improve itself by people using it. In other words, it is within possibility that this can be programmed to automatically perform self-organizing actions on its data to improve

its data structure across and by all orders of complexity and domains. This is my expectation.

Axiomatization

Axioms are important because they describe the rules of action for an entity performing an action. In this case, the entity is the recursive architectonic and its FPC. To the best of my abilities, I attempted to outline an axiomata, a set of axioms that could act as a springboard for conversion of recursive architectonics into a computational environment. Here, the recursive architectonic is a synthesis of complexity and qualia, so therefore an axiomatization of both was required. For complexity, this meant order and transition. For qualia, this meant qualia abstractions assembled by order and transition, and vice versa.

For complexity axioms, this required order axioms and transition axioms. Commons had recently updated his axioms for orders before I began this thesis (Commons, Gane-McCalla, Barker, & Li, in press). Here, I took them further, and classify them as linear order axioms. I preserved them intact, and compensated for their lack of nonlinearity by creating a second set of axioms, nonlinear axioms, that reflected Commons' axioms in every way, except that the nonlinear axioms described MPC's diagonal complexity's nonlinearity. Both sets of axioms are taken here as equally valid, but here I propose that they yield the most integrated approach when used together. The qualitative actions in axiomatic expressions can be expressed in different language symbols, and here I followed Commons' mathematical combinatorics with the nonlinear axioms.

Transitions between orders, however, were a different matter. As it turned out, the axioms for transitions were FPC-like, though not in a recursive architectonic modeling: they were content-free multinamic, recursive, and nested, while indexing was the content of the entities, in that in certain circumstances, an entity can index itself limited by the constraints of the order of its complexity for which indexing is performed. In the context of the MPC, the states of the phase transitions, are hypothesized here as fractal representations of order 0 phenomena inherited in all increasing complexities. My demonstration (Barker, in preparation) of the universality of the phase transitions across scales and domains is enough evidence without need for metaphysical debates. The qualitative actions of phase transition sequences can be expressed in different language symbols, and here I chose to use logic for its aesthetic.

Qualia required a notational rendition of qualia abstractions. I axiomated each qualia abstraction, and as mentioned above, used nests with nested fractal matrices to capture the nonlinear n-dimensions. I employed multiple mathematical notations for what seemed best for a function since I had developed a means to bridge them into each other consistently via indexing. I used fractal propositional calculus, category theory, and Laplacian operations to representing nesting. I used Cartesian coordinates and linear algebra for capturing the n-dimensions throughout them.

Summary

Methodologically, it became apparent early on that there was no such thing as a best notation for the quatypl qualia. The quatype properties were inherent in all notation, so that a best signification and syntax schema were relative to the function of a given use. In essence, reality is the best example of itself. The discussion above and the

results of using these approaches indicate the congruence of the FPC development with “how reality works,” while maintaining a consistent means of integrating schema that challenge its representation so as to improve itself inherent in its architecture as part of its process. By taking this approach, the methods discussed above unexpectedly helped me contextualize universal calculability in a greater recursive architectonic scope, as discussed in the following chapter.

CHAPTER V

RESULTS

The purpose of this chapter is to report the results of my effort to develop fractal phase calculus (FPC) for its expression in computer-usable language. These results indicate how I bridged extremely high-level abstraction into a language computers can understand. I report the FPC notation and axioms for operationalization in software coding.

At a high level, and as discussed in Chapter IV, the results of this work convey the appearance of the paradoxical relative formlessness of the universal formula, and my implementation of that understanding into application. In this chapter I give a more detailed description of the formlessness as briefly outlined in the previous chapter, articulate the current iteration of the set of axioms and theorems that accommodate and span all qualia and complexity, and then give an example of application.

The Relative Formlessness of the Universal Formula

The debate between relativity and universality is a millennia-spanning quarrel. The relativistic perspective generally states that there are no universals, that everything is relative. This is a contradiction, because relativity is itself presented as a universal. The universalist perspective generally constrains universality down to absolute truths. This is a contradiction, because as knowledge progresses, what was once thought to be absolute

truth must be replaced with more accurate renditions. Commons and Richards (2002) showed quite clearly that relativity and universality map onto deconstructive and constructive processes of transition dynamics between orders of complexity, which seems an adequate explanation for why relativity and universality are scale-free across stages of behavior, and we can never do away with either; they are no more problems to be overcome than a repeating process that enables development is a problem. Gödel's theorems and the Turing-Church thesis built upon it are inadvertent mathematical demonstrations of the natural cycle of human improvement that drives evolution forward.

Universality and relativity are both true; universality is a synthesis of the relativity of actions by means of their shared properties in a coordinated manner, and by means of the new synthesis, a relativity to the greater scope of qualia and complexity to which it belongs. Vertical complexity describes that process of the development of action at increasingly universal abstractions of integral properties across all qualia and complexity, and horizontal complexity describes the relativity of property differentiation across all qualia and complexity. Diagonal complexity reveals the fractal nature of recurring vertical complexity differentiated at tiers (scales) of qualia assembly with increasing resolution according to the capacities of an observer, capacities that have been developed out of latency along all three n-dimensions, with respects to what the constraints of the observer's organismic architectures and processes allow.

I suggest, based on my personal observation of a very limited number of people who appear to exhibit performative-recursive behaviors (see Ross, 2008b; Ross, Commons, Li, Stålné, & Barker, in press), by the time a person begins a transition into order 15 performative-recursive (also called metacross-paradigmatic), one has

differentiated and integrated together the entirety of one's conceptual representation of reality into some kind of unified framework. The only order of phase complexity available to perform is to continue differentiation and integration, to make modifications by virtue of them, and to improve that integrated architectonic representation as the individual goes through time and gathers more experiences. These events create a series of horizontally complex integral architectonic versions of reality for which their only comparison is their earlier iterations, that is, cross-paradigmatic actions across qualia domains and abstractions. But when the alternative versions of one's own integral architectonic and or that of others are merged, the resulting reorientation of parts creates a cascading ripple into other parts, all of which are governed by coordinative action of the whole, leading to an effect where the whole is both affected by and effecting the parts as much as the parts are affected by and effecting the whole (e.g., as explained by Alexander & Globus, 1996). The key property here is that the whole is coordinated consciously by the individual, not simply by autonomic or accidental actions. These performative-recursive behaviors, I propose, transform the integral architectonic process into a recursive architectonic one. The recursive architectonic is, I propose, the qualia assembly of performative-recursive complexity.

I propose this above synthesis, in part, from the multiplicity of performative-recursive refinements of integral architectonic frameworks, potentially an order 15 action, from the 84 versions of SHIM I have performed over several years. I recognize those numerous recursive iterations as a universal processual architecture that threads throughout the entirety of all the parts and the whole itself, the universal unit, which I conjecture to be phase complexity order 16. Fractal phase calculus was my attempt to

axiomatize the principles of performative-recursive behaviors, but the properties are so profoundly abstract, that they have no form in and of themselves, i.e., *forma* is the qualia of form, and to represent it in mathematical form is three lesser magnitudes of qualia abstraction.

On the one hand, my findings mean FPC is universally accessible relative to the constraints of whatever inherits it. For example, I can communicate it here in English, and by using the logical, mathematical, and propositional calculus notation below, these can be understood by means of the qualia assemblies and phase complexity available in the reader. But on the other hand it is beyond not just the tangible but even the intangible and therefore I cannot capture it in its purity, though paradoxically it is present everywhere in its purest way.

The Axiomatization

One can axiomatize the qualia abstractions and phase complexities, but they are downwardly assimilated simulations which can be done in any language with any notation, insofar as the signification of the symbols, rules of syntax, and pragmatic syntheses of them are made clear. This is because these principles are inherent in all qualia abstractions and phase complexities. This simulation can be done at any scale, in any direction, with any qualia abstraction, with any qualia assembly, with any number of parts, by any phase transition, at any phase order. It is not just about the content of the processual architecture that is important, but the processual architecture itself. This is because in the recursivity, the processual architecture participates as the content. Below I give an in-progress rendition of the axiomata, followed by a simple example of it in action.

Table 3. Sign index.

Type	Symbol	Description
Mathematical Signs	∞	Infinity
	$=$	equivalent to
	\approx	approximate image of
	$\bullet\infty$	exact image of
	Φ	morphism, mapping onto or into
	Σ	Summation
	∇	Gradient
	Δ	change in
	\geq	greater or equal than
	\circ	Concatenation
	$()$	round bracket, for specification of distribution
	$\{ \}$	curly bracket for sets
	$[]$	square bracket designates a tuple, and orderly concatenation of elements
$x \int_0^n$	functional integral, (0, 1, 2...n); each integer designates a function of x. Sequential when designated by square bracket.	
$f(x)$	function	
Logical Signs	\forall	All
	\exists	there exists

Type	Symbol	Description
	\neg	negation, not
	\vee	or
	\wedge	and
	\cap	intersection, intersected by
	\cup	unification, unified with
User Defined Signs	0x	left superscript, tier of x. Alternate n-dimensions/divergences specified as ${}^{0.0}x$
	x_0	right subscript, instance of x. Alternates of same instance specified as $x_{0.0}$
	\mathbb{Q}	completeness of qua assembly
	\propto	Para-completeness signifier, channel of transference
	●	Static
	⊙	Dynamic
	⊗	Multinamic
	⊗	<i>Etherealis</i>
	⊕	<i>Corporealis</i>
	⊛	<i>Psycherealis</i>
	↪	Recursivity

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Table 4. Spectrum of Human Imagination Model, Qualia Axioms.

Abstraction	Axiom Description	Notation
Quatypal	Axiom of Multinamism: All qua is static and dynamic simultaneously with flux, which is the equivalent of multinamic.	$\{\bullet, \odot\} = \odot$
	Axiom of Static: When in equilibrium, a multinamism may be expressed as static	$\bullet = \odot$
	Axiom of Dynamic: When in action, a multinamism may be expressed as dynamic	$\odot = \odot$
	Axiom of Completeness: All qua is complete	$\forall Q = Q\alpha$
	Axiom of Incompleteness: All qua is incomplete	$\forall Q = \alpha Q$
	Axiom of Para-completeness: For all qua, a qua is complete, and it is incomplete simultaneously, infinitely.	$\alpha Q\alpha = \infty$
	Axiom of Differentiation: All qua that is divided, divides into equal to or greater than one component	$\forall Q / Q (n \geq 1)$
Axiom of Integration: A qua that is combined out of components, is combined by equal to or greater than one component, with equal to or greater than one relation	$(\sum \geq 1(Q)) \cap (\sum \geq 1(Q)) \cup \forall Q$	
Deitypal	Axiom of Qua Self-Image: All qua is an image of all qua	$\forall Q \leftrightarrow \forall Q$
	Axiom of Qua Self-Reference: The completeness of a qua is self-referential, leading to incompleteness which returns its incompleteness to itself, returning to completeness	$\forall Q \cup = Q$
	Axiom of Qua Existence and Non-existence: All that exists and does not exist is infinitely qua	$\forall \exists \infty = Q, \forall \neg \exists \infty = Q$
Archetypal	Axiom of Intangibility: All qua of non-tangible equilibria and actions can be expressed at least approximately through a	$\forall Q \approx \otimes(\varphi)$

Abstraction	Axiom Description	Notation
	morphism	
	Axiom of Tangibility: All qua of physical equilibria and actions can be expressed at least approximately through a morphism	$\forall Q \ni \oplus(\varphi)$
	Axiom of Quasi-tangibility: All qua of mental equilibria and actions can be expressed at least approximately through a morphism	$\forall Q \ni \otimes(\varphi)$
Coordinate mapping	Axiom of Qualia Gradients: Through any and all permutation of any and all axioms, qualia gradients are constructed	$\nabla Q \int_0^8 = 0 = [\nabla Q0, \nabla Q1, \nabla Q2, \nabla Q3, \nabla Q4, \nabla Q5, \nabla Q6, \nabla Q7, \nabla Q0] = \nabla Qn,$ $\nabla Q8 = \nabla Q0 = \text{quatype}$ $\nabla Q1 = \text{monotype}$ $\nabla Q2 = \text{nuotype}$ $\nabla Q3 = \text{polytype}$ $\nabla Q4 = \text{omnitype}$ $\nabla Q5 = \text{metatype}$ $\nabla Q6 = \text{archetype}$ $\nabla Q7 = \text{deitype}$ $\nabla Q8 = \nabla Q0 = \text{quatype}$
	Axiom of Monotype Construction: Monotypes are constructed from any quatypal assembly	$\nabla Q8\varphi\{\nabla Q\{n,n\} \sum \geq 2\} \cup \nabla Q1\varphi\{n,n\}$
	Axiom of Nuotype Construction: Nuotypes are constructed from any quatypal assembly	$\nabla Q1\varphi\{\nabla Q\{n,n\} \sum \geq 2\} \cup \nabla Q2\varphi\{n,n\}$
	Axiom of Polytype Construction: Polytypes are constructed from nuotypal qualia assembly	$\nabla Q2\varphi\{\nabla Q\{\forall\{n,n\}\} \cup \nabla Q3\varphi\{n\}$
	Axiom of Omnitype Construction: Omnitypes are constructed from polytypal qualia assembly	$\nabla Q3\varphi\{\nabla Q\{\forall\{n\}\} \cup \nabla Q4\varphi\{n\}$
	Axiom of Metatype Construction: Metatypes are constructed from omnitypal qualia assembly	$\nabla Q4\varphi\{\nabla Q\{\forall\{n\}\} \cup \nabla Q5\varphi\{n\}$
	Axiom of Archetype Construction: Archetypes are constructed from metatypal qualia assembly	$\nabla Q5\varphi\{\nabla Q\{\forall\{n\}\} \cup \nabla Q6\varphi\{n\}$

Abstraction	Axiom Description	Notation
	Axiom of Deitytype Construction: Deitytype is constructed from archetypal qualia assembly	$\nabla Q6\phi\{\nabla Q[V\{n\}\} \cup \nabla Q7\phi\{n\}$
	Axiom of Quatype Construction: Quatype is constructed from deitytypal qualia assembly	$\nabla Q7\phi\{\nabla Q[V\{n\}\} \cup \nabla Q8\phi\{n\}$
	Axiom of Monotype Qualia Gradient Coordinates: Any qua may begin a new assembly starting at cardinal 0 with n-dimensions	n-dimensions = [0...n] = x, [0...n] = y
	Axiom of Nuotype Qualia Gradient Coordinates: Any qua may begin a new assembly starting at cardinal 0 with n-dimensions	n-dimensions = [0...n] = x, [0...n] = y
	Axiom of Polytype Qualia Gradient Coordinates: All nuotypal qualia assemble into one or more orderly coordinates of polytypes	if <i>organism</i> = 0, then [[-17...0...9] = x, [-16...0...16] = y] = [n,n]
	Axiom of Omnitype Qualia Gradient Coordinates: All polytypal qualia assemble into orderly coordinates of omnitypes	if <i>organizatio</i> = 0, then [-17...0...9] = n-dimension
	Axiom of Metatype Qualia Gradient Coordinates: All omnitypal qualia assemble into orderly coordinates of metatypes	if <i>organon</i> = 0, then [-6...0...3] = n-dimension
	Axiom of Archetype Qualia Gradient Coordinates: All metatypal qualia assemble into orderly coordinates of archetypes	if <i>corporealis</i> = 0, then [-1, 0, 1] = n-dimension
	Axiom of Deitytype Qualia Gradient Coordinates: All archetypal qualia assemble into an orderly coordinate of deitytype: mouth, body, tail	if <i>divus</i> = 0, then [-1, 0, 1] = n-dimension
	Axiom of Quatype Qualia Gradient Coordinates: All qualia assembly of deitytype assemble into an orderly coordinate of quatype: dynamic, multinamic, static	if <i>multinamic</i> = 0, then [-1, 0, 1] = n-dimension

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Table 5. Phase Complexity Axioms.

Complexity	Axiom Description	Notation
Linear Phase ^a	Linear Complexity Axioms Axiom of Order: The orders of complexity are constructed in a linear way	if $a > b$, then $\varphi(a) > \varphi(b)$
	Axiom of Transitivity: The linearity of complexity is transitive	if $a > b$ and $b > c$, then $a > c$
	Axiom of Linear Chain: Linear complexity does not increase if two actions or more are of the same order of complexity	$\varphi(a \circ b) = \max(\varphi(a), \varphi(b))$ if $\varphi(a \circ b) = \varphi(b \circ a)$
	Axiom of Linear Coordination: Linear complexity stratifies if one action is more complex than another action	$\varphi(a \circ b) = \max(\varphi(a), \varphi(b)) + 1$ if $\varphi(b) = \varphi(a)$ and $\varphi(a \circ b) \neq \varphi(b \circ a)$
Nonlinear Phase	Axiom of Disorder: The orders of complexity are constructed in a non-linear way	if $a > b > a$, then $\varphi(a) > \varphi(b) > \varphi(a)$
	Axiom of Non-transitivity: The non-linearity of complexity is non-transitive	if $a > b$ and $b > c$, and $c > a$, then $c \geq a$ and $b \geq c$
	Axiom of Non-linear Chain: Non-linear complexity does not increase if two or more actions are of the same order of complexity	$\varphi(a \circ b) = \max(\varphi(a), \varphi(b))$ n-dimension if $\varphi(a \circ b) = \varphi(b \circ a)$
	Axiom of Non-linear Coordination: Non-linear complexity stratifies if one action is more complex than another action	$\varphi(a \circ b) = \max(\varphi(a), \varphi(b))$ n-dimension + 1 if $\varphi(b) = \varphi(a)$ and $\varphi(a \circ b) \neq \varphi(b \circ a)$
Phase Transition	Axiom of Phase Transition Universality: All qua is expressive of phase transitions	$\forall Q = \Delta T(s)$
	Axiom of Phase Transition Fractal: All qua are a fractal of phase transitions	$\forall Q = \Delta T(s) = \Delta T(\Delta T(s))$
	Axiom of Phase Transition Algorithm: All entities that act, act at some order of phase complexity, and the actions will change states; at minimum, a	Barker (in preparation)

Complexity	Axiom Description	Notation
	destabilization of temporary equilibrium. If conditions permit more than just a destabilization, phase transition may occur. If conditions permit, three phase transitions of phase complexity may be enacted. Conditions refer to constraints of the entities internal and external environment architectures and processes.	
	Axiom of Static Transition: A temporary equilibrium can be mapped onto a qua.	$\varphi 0 (\varphi(^0Q_1))$
	Axiom of Dynamic Transition: The phasic interaction between two or more staticisms can be mapped onto a qua.	$\varphi 1 (\varphi(^0Q_1) \odot \varphi(^0Q_2))$
	Axiom of Multinamic Transition: The phasic synthesis between two or more staticisms can be mapped onto a qua.	$\varphi 2 (\varphi(^0Q_1) \odot \varphi(^0Q_2)) = n\text{-dimension} + 1 = \varphi(^1Q_1),$

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^a Mathematical renditions of what here I term “linear axioms”, are from Commons, Gane-McCalla, Barker, & Li, The model of hierarchical complexity as a measurement system. *Journal of Adult Development* (in press).

This axiomata essentially describes a mathematical and logical representation of the integral architectonic, and the deitypal and quatypal functions enable the model to contain and reorganize itself as a recursive architectonic. The n-dimensions index the input, and the content of the n-dimensional nesting has access to reorganize itself by means of the process functions described by the architectonic itself.

Example of Application

I used three notation types: long-handed, medium-handed, and short-handed. Long handed notations detailed explicitly the entirety of a fractal phase calculation, but the formulae were so massive that computer software especially designed to write

mathematical notations could not handle the size of these formulas. The short-handed notations are extremely simplified, but would be unreadable to anyone without a clear understanding of the subject matter. The medium-hand approach to notation is a pseudocode, which I use here to present an example of application. Medium-hand is middle ground between these two polarities. It includes descriptive mapping to quasi-numerical indexing, and then uses them. In order to demonstrate the formulae, I give an example of the development of a molecule through qualia abstraction and phase complexity. In addition, this demonstrates that qualia abstraction and phase complexity are not just applicable to human experience and action only. I refer the reader to the Appendix to see the locations of the coordinates described below.

Here, I apply the aforementioned signs and axioms as I describe a formulaic representation of a molecule. The function of this example is to demonstrate the scale-free nature of the recursive architectonic properties. From bottom-up, a molecule can be mapped as a nuotypal qualia assembly of *chemistry* by means of any given indexed ontological schemata mapping to polytypal qualia magnitude coordinate $\{-3x, -7y\}$, of the omnitypal organizatio coordinate $\{-3x\}$, of the organon metatype at coordinate $\{-1x\}$, of the corporealis archetype coordinate $\{0x\}$, of the deitype coordinate $\{0x\}$, of the quatype $\{0x\}$. This is an implementation of SHIM axioms, and yields a nested qualia assembly coordinate of:

$$\nabla Q_8 \varphi \{0x\} (\nabla Q_7 \varphi \{0x\} (\nabla Q_6 \varphi \{0x\} (\nabla Q_5 \varphi \{-1x\} (\nabla Q_4 \varphi \{-3x\} (\nabla Q_3 \varphi \{-3x, -7y\})))))) \quad (1)$$

where each lesser abstraction is a differential propositional calculus function in which it is nested, and an integral propositional calculus for that which it nests. This formula can be assigned in a simplified manner as a variable or number, which enables a direct conversion into mathematics as shown below. This lends some short-hand to the later formulae. The reason for italicization of quasi-arbitrary numerical assignments is to differentiate the numerical assignments of conceptual representations from the rest of the math.

$$I = \nabla Q_8 \varphi \{0x\} (\nabla Q_7 \varphi \{0x\} (\nabla Q_6 \varphi \{0x\} (\nabla Q_5 \varphi \{-1x\} (\nabla Q_4 \varphi \{-3x\} (\nabla Q_3 \varphi \{-3x, -7y\})))))) \quad (2)$$

This formulae may also be written in the following pseudocode syntax:

$$\begin{aligned}
 I = \nabla Q_8 \varphi \{0x\} (& \hspace{15em} (3) \\
 \nabla Q_7 \varphi \{0x\} (& \\
 \nabla Q_6 \varphi \{0x\} (& \\
 \nabla Q_5 \varphi \{0x\} (& \\
 \nabla Q_4 \varphi \{0x\} (& \\
 \nabla Q_3 \varphi \{0x, -7y\} & \\
) & \\
) & \\
) &
 \end{aligned}$$

)
)

In order to map the development of a molecule, one must account for the molecular interior static and dynamic in SHIM terms, and the molecule interior entities and their actions in MPC terms.

The molecule is composed of entities, atoms. The atoms are nuotypal qualia assemblies of chemistry, and can be mapped to polytypal coordinates $\{-5x, -7y\}$, omnitypal coordinate $\{-5\}$, metatypal coordinate $\{-1\}$, archetypal coordinate $\{0\}$, deitype coordinate $\{0\}$, and quatype coordinate $\{0\}$. Below, atoms are given the quasi-numerical assignment the italicized 2.

$$2 = \nabla Q_8 \varphi \{0x\} (\tag{4}$$

$$\nabla Q_7 \varphi \{0x\} ($$

$$\nabla Q_6 \varphi \{0x\} ($$

$$\nabla Q_5 \varphi \{-1x\} ($$

$$\nabla Q_4 \varphi \{-5x\} ($$

$$\nabla Q_3 \varphi \{-5x, -7y\}$$

)
)
)

)
)

The interactions among the atoms are, in the nuotypal qualia assembly of chemistry, called atomic or intramolecular forces. These forces are inherited as a sub-action of the assembly's parts, nested functions culminating in the following coordinate mapping as:

$$3 = \nabla Q_8 \varphi \{1x\} (\tag{5}$$

$$\nabla Q_7 \varphi \{0x\} ($$

$$\nabla Q_6 \varphi \{0x\} ($$

$$\nabla Q_5 \varphi \{-1x\} ($$

$$\nabla Q_4 \varphi \{-4x\} ($$

$$\nabla Q_3 \varphi \{-4x, -7y\}$$

)
)
)
)
)

Here I have assigned the italicized number *1* to molecule, *2* to atom, and the italicized number *3* to intramolecular force. The qualia assembly of a molecule requires

two or more atoms, and the Axiom of Integration holds. Each atom can be assigned an instance of 2, differentiated by further nesting of nuotypal coordinates and the index called to represent it as symbols. Any two given atoms in any given molecule that is given molecular status in the chemistry nuotype can be given additional symbolic assignments. In other words, this method means one could map the Table of Elements (e.g., one could use atomic numbers) and all molecules that are constructed from them, with recursive architectonic mappings. For any nuotypal qualia assembly schema for the representation of each atom, one can map them into SHIM, here assigning atoms the italicized number 2, and using the right subscript sign to indicate an instance of the prototype, whereas 2 and 2_0 indicates the prototype itself, we may call the first instance of an atom 2_1 , and the second instance of an atom 2_2 . The number 3 has been assigned to the intramolecular force. The forces of intramolecular force are sub-actions, and the fractal nesting of the four forces can be mapped to their polytypal source coordinates at $\{-3x, -11y\}$. I introduce this application without further discussion simply to show these entities are, in fact, accounted for.

The gestalt of the molecule occurs by means of phase transition. The phase transitions are scale-free actions that are performed by and transpire between entities where the axiom of phase algorithm holds. Phase transitions are defined in Chapter I, and the related axioms are described in Table 5. Any entity (i.e., static) and action (i.e., dynamic) can be mapped into phase transitions.

Below, a description of molecular construction is described using SHIM qualia mappings and orders of phase complexity. I begin by describing the two orders of phase complexity at work in molecular construction. Phase order 1, analog, describes two or

more actions or entities producing analogic values by means of intrinsic binary behavioral properties of the prior order of action, 0. Order of action 2, automata, describes two or more actions or entities producing class-conditional properties by means of the configurational constraints intrinsic between analogic behavioral properties of order of action 1.

The interior order of phase complexity of atoms as entities reaches a maximum limit of phase complexity at order 2, automata. However, the construction of a molecule occurs at action order 1, analog. This is because atoms undergo combinatory phase transition actions with other atoms in the qualia assembly of a molecule, so that there is a diagonal n-dimension where the new entity, the molecule, assembles by means of a lower action order than the max action order of the parts. This is an example of diagonal n-dimensional phase complexity; the magnitude (scale) of qualia assembly increases vertically from polytypal coordinates of the atom $\{-3x, -8y\}$ to molecule $\{-3x, -7y\}$ where the axiom of order holds, but in doing so, the phase order decreases from ∇O_2 (order of phase complexity 2) to ∇O_1 (order of phase complexity 1) at the increased SHIM scale that has been produced, $\{-7x\}$.

Here, vertical transition of phase complexity is represented, with diagonal complexity linking the actions as the basis for qualia assembly. Here, the numbering of the gradient " ∇ " of phase order 0 "O0" is written ∇O_0 , and the phase transition ∇T gradation from 0-2 are the derivative of the function ∇O_0 . Here, I will iterate that italicized numbers are numerical assignments on conceptual representations that are being described as they undergo phase transitions of phase complexity. Below is a demonstration the automation of atoms assembling into a molecule.

$$\nabla O_0(\nabla T_0) \varphi 2_1 \quad (6)$$

Protons, neutrons, and electrons are in temporary equilibrium at phase order 2, automata, and the atom itself has reached max limit ∇O_2 . The automata's specific properties determines its constraints for actions with external entities, and external combinatorics begins anew at the new scale as ∇O_0 .

$$\nabla O_0(\nabla T_1) \varphi (2_2 \odot (3^{\sum k \geq 1})) 2_1 \quad (7)$$

The exchange of forces between 2_1 and 2_2 .

$$\nabla O_0(\nabla T_2) \varphi (2_2 \odot (3^{\sum k \geq 1})) 2_1 = (\nabla O_1(\nabla T_0) \varphi 1 \quad (8)$$

The atoms form a temporary equilibrium, a molecule.

The vertical phase complexity from action order 0 binary to action order 1 analog is the foundation for the combinatoric construction of the qualia assembly of a molecule, with the index preserved in the indexed calculi nesting.

Next, another combinatoric vertical phase complexity follows, producing the phase complexity action order 2, automata, which describes the newly formed molecule defining itself to its exterior by means of its internal entities, actions, and ensuing constraints by means of them. The vertical increase in complexity of a molecule as action order 2 is described below.

$$\nabla O_1(\nabla T_0) \varphi I \quad (9)$$

Molecular temporary equilibrium

$$\nabla O_1(\nabla T_1) \varphi (I \odot (\nabla Q_4 \sum_{k \geq 1})) \quad (10)$$

The molecule exhibits its forces on its environment and has forces exerted upon itself with properties of the molecule emergently defining itself to itself and environment, the molecule tests as a stable temporary equilibrrious substance amidst the chaotic forces.

$$\nabla O_1(\nabla T_2) \varphi (I \odot (\nabla Q_4 \sum_{k \geq 1})) = \nabla O_2(\nabla T_0) \varphi I \quad (11)$$

The molecule achieves temporary equilibrium at automata phase complexity. Its means of interacting with the environment is stable.

Here, I only mean to give a general example of how recursive architectonic synthesis of SHIM and MPC works. The finer grained calculi for phase states within the phase transitions are explicitly demonstrated elsewhere (Barker, in preparation). The number assignment allows one to convert any processual architecture of qualia and complexity to a numbering index. Each division and sub-division represents a quasi-arbitrary numbering scheme. By following the quatypal axioms, the formless universal formula, here shown to be convertible into Church-Turing computability, allows a universally infinite indexing architecture as schemas are added and operated with, while allowing a quasi-arbitrary indexing that is flexible enough to serve any purpose relative to its function. Since the mappings are numerical, data can be stored as a series of numbers. Any and every unit here in the example exhibits quatypal and deitypal

properties through n-dimensions embedded in its existence, while exhibiting as MPC the phase transitions through n-dimensions in any and all relations to any other thing, at any particular given action order it inherently expresses.

Summary

Here I have demonstrated that the qualia and complexity inherent in a conceptual representation can be mapped into FPC notation, and be reduced to mathematical and logical operands and operations of numerical form.

Static, dynamic, and multinamic are reflected as atoms, intramolecular forces, and molecules. Recursivity is described as the recurring properties of the phase transition and action of phase complexity processes through the diagonal complexity across qualia magnitudes. The vertical complexity is mathematical captured in qualia abstractions, qualia magnitudes, phase transitions, and orders of phase complexity gradation. Indexing is used in all the coordinate mappings of qualia abstractions and phase complexity, reduced to simple integers.

The nuotypal qualia abstraction allows one to clearly segment an index of a representational ontology in a conventional domain of knowledge, without making judgments on its truth-value. Since the index is preserved in a quasi-arbitrary numbering scheme, and since the numbers are preserved in nested dependencies, it follows that any alteration to recursive architectonic architectural and processive modeling, would not alter the segmented indexes, only add an additional nested index of change over time, thus shifting the relative mapping of segmented nuotypal indexes according to the architectural and processive modeling changes. The quasi-computational representation

is just one way of describing an architecture and process, as there is no best form of representation.

CHAPTER IV

DISCUSSION

In this discussion chapter, I discuss three topics that follow from the previous chapters. In the first section, I continue from Chapter V with a brief discussion of performative-recursive architectonic operations. In the second section, I revisit the meaning of fractal phase calculus in light of the recursive architectonic synthesis. In the third section, I discuss how my work paves the way for advancement in computational emulation of human behavior. In the fourth section, I discuss how recursive architectonic software could serve the human species. In the fifth and final section, I close with a brief thesis summary.

Performative-Recursive Architectonic Operations

In the previous two chapters, mapping phase complexity and its qualia assembly content to computable symbols was both described and demonstrated. Thus, it can be said that I accomplished my primary objective for this thesis, to design a functional fractal phase calculus (FPC) prototype. However, although the performative-recursive action and recursive architectonic have been described, they have not been demonstrated in the earlier given computational notation. While that step is beyond the scope of this thesis, it is a fair question to wonder what that next step would involve.

Thus, here I provide some discussion about what it will take to describe this in computational notation. The approach I take is to give an example of the operations I had to perform when something arose that changed the information in SHIM's architectonic. In an earlier integral architectonic rendition of SHIM 84.6, the qualia assembly scale of the international scale (polytypal tier 6) was described to assemble a larger scale of qualia as the hemispheric scale (polytypal tier 7), but these were resolved to be one and the same tier when I recognized that the relationship between national relations were not necessarily linked by space. This simple change caused vertical and horizontal rippling effects for how qualia assembly domains and scales of content were organized and named, and caused a shift across the entire model. The changes were vertical because all the descriptions for the qualia assemblies of polytypal tiers 5, 6, 7 and 8 had to be reconstructed to better fit and speak for the qualia assembly 6 and 7, which were combined to 6. These changes were also horizontal because all polytypal fractals share the tiers being effected and affected, and therefore each of the polytypal fractals must match in symmetry of magnitude, so that the change was to be reflected across all polytypal fractals of qualia assembly.

Once polytypes were synthesized abstractions from their newly reoriented nuotypal contents, then a check had to be made if all the omnitypes held true for the inherited and integrated polytypal fractal differentiations that had been made. That process of checking through the SHIM model had to extend with metatypes, archetypes, deitype, and finally quatype to question and validate the universal property across the changes. The whole was effected and affected by the changes of the parts, and parts were effected and affected by the changes in the whole. Each change in each ambit of qualia

effected itself through the transformative change it compelled on its surroundings, reconditioning a new context of the whole. This applied example of actual work that had to be performed on all relationships in the model by the model was the performative-recursive dynamic in action, via all these relational interactions. While I realize this language is abstract, it genuinely describes the tasks and phenomena I experienced.

In order to demonstrate performative-recursive operations on the integral architectonic through computational symbols, I would have to render the process as described above, with all its nested fractal sub-actions in fine-grained detail using the FPC axioms as a frame. The process would have to be performed upon the entire breadth of relevant human conceptions across all domains of knowledge—those of which I was aware during the operation, that is—which pertained to these polytypal fractal tiers being transformed. Such a computational demonstration doesn't strike me as difficult, but it would be very time consuming and for the scope and purpose of this thesis, it is unnecessary to undertake as an academic exercise.

The Meaning of Fractal Phase Calculus, Revisited

It seems to me quite fitting to describe SHIM as fundamentally descriptive of architecture, and MPC as fundamentally descriptive of process. Though they each exhibit multinamism, recursivity, n-dimensions, and indexing in their modeling, I propose that the reason why these two models synthesize so well is because they are themselves recursive iterations of the two multinamic properties of integral architectonic semblance. The definition given in Chapter II for FPC still holds, but the work done in this thesis has reinforced that definition in several ways.

In terms of multinamism, MPC's description of entities (static) and actions (dynamic) can be synthesized with SHIM's description of static and dynamic. In terms of n-dimensions, both share in common vertical, horizontal, and diagonal dimensions, their meaning and functioning fundamentally interchangeable as well. Further, each share identical descriptions of fractal scales along those n-dimensions, and therefore the qualia magnitudes and orders of phase complexity can be combined into a single abstraction, *phase architecture*. Similarly, qualia assembly and phase transitions may be combined into a single abstraction as well, *phase process*.

In this context, *phase architecture* becomes an appropriate replacement for *static*, and *phase process* becomes an appropriate replacement for *dynamic*, and the term *phase* becomes synonymous with *multinamism*. The recursivity and n-dimensions are foundations for using the term *fractal*, and the indexing the foundation for using the term *calculus*. Thus, *fractal phase calculus* as a term I conceived of earlier in the thesis remains a correct term to describe what it is intended.

Towards Advancing Computational Emulation of Human Behavior

As discussed in Chapter III, Gödel's theorems describe downwardly assimilated properties of human experience in general. Tarski was on to this, but the relationship between mathematics, computation, and human behavior could not be made until Ross identified the fractal nature of transition dynamics, and I was able to make the correlation across these domains during this thesis.

I propose that Gödel's completeness theorem is fundamentally rooted in the fractal property of temporary equilibrium (Barker, in preparation), a scale-free property at all orders of phase complexity. This is because completeness is conditioned on the

consistency formed from the coordination of entities and or actions from a lower order of phase complexity: unification is coordinated. Just as Kant spoke of the architectonic as a consistency between cognitions, similarly true for any completeness. Gödel's incompleteness theorems are fundamentally rooted in the fractal property of the other phase transitions, also scale-free properties, but between orders of phase complexity: relativism yields incompleteness. This is because a new entity or action is presented that destabilizes the previous completeness, and in order to maintain consistency, it requires a coordination of the disparateness presented by the destabilization.

Whereas Gödel's theorems may have been long considered to be a perplexing problem to be solved, here I propose that what underpins Gödel's theorems is a natural process of evolution. It is not so much a problem to be solved or a paradox to be untangled. I propose that the orders of phase complexity and phase transitions are scaffolding of development to be climbed, and a recursive process that drives nature forward.

This means that it is now possible to accomplish what Leibniz originally intended with his *characteristica universalis*, and what Turing intended with his notion of intelligent machines to its fullest potential. While it can be said that computational models already exist that, in a sense, obey multinamism, recursivity, n-dimensions, and indexing, the recursive architectonic makes way for organizing human intelligence into machines in a way previously impossible by providing (a) an integral architectonic for coordinating all the contents of actions, (b) an integral architectonic for coordinating all the actions of contents, (c) a process by which (i) new data and methods that destabilize their completeness are met with (ii) a performative-recursive process that guides the

differentiation into a consistent integration by (iii) means of having a mapping architecture by which the qualia and complexity of the difference can be identified and correlated. This is because FPC units of data and method are, from the beginning, embedded with universal properties, *situated in their scope*, the human experience of qualia and phase complexity, thereby enabling integration up to the qualia and complexity magnitude of FPC.

Functions of Recursive Architectonic Software

In this section I discuss some of the potential functions of recursive architectonic software, which are the reasons I made the effort to design an FPC. In the current state of academe, the general process of knowledge creation entails individuals and groups performing various methods of knowledge creation, and presenting their data in literature, presentations, and information databases. Though this means of sharing information has endured for ages and in my opinion should be preserved intact, there are inherent problems with this process.

The knowledge creation process involves seven kinds of action to be mentioned here: (a) disseminating information, (b) unifying dissemination feedback, (c) supporting the integration and synthesis of differentiation across scales and breadths of participation, (d) identifying uncharted knowledge frontiers, (e) modeling dynamic knowledge, (f) empowering communication, and (g) the creation of a globally participatory theory of everything.

First, with respect to disseminating information, although many academic papers go through rigorous peer review processes, once the paper is released to the subscription-only databases or publicly-accessible internet venues, the information does not always

find its way to the individuals and groups that would benefit from knowing about it. Designing a recursive architectonic software would be aimed to serve this purpose, allowing people to find information most relevant to their interests. This would be possible because even though people use different terms across multiple fields, the organization of information in the recursive architectonic architecture would be done by means of its qualia and complexity, essentially bypassing the communication problem. The relationships between information would not be organized by arbitrary syntactical strings which motivate modern search engines, but rather, information would be organized by the natural architectonic relations of the properties of the topics.

Second, even in the case where the information is accessible by means of broad dissemination, support and criticism with rationale are largely dispersed sporadically across multiple information architectures. This leads to a disorganized integration of differentiation, where multiple discussions form between multiple individuals and groups, disjointed from each other. The total range of support and criticism is not bound together into a unified architecture. For example, while it is true that Wikipedia has a talk page, this page is for discussing improvements to the article, and not for general discussions of the subject. There simply does not exist software that enables global discussion in any kind of unified way. Creating online recursive architectonic software would serve not just as a user-created knowledge database, but also give a framework for these global discussions, which are essential discussions for knowledge to pick up the pace of its evolution.

Third, whereas individuals and groups engage in discussions, individuals and groups often fracture and/or dissolve by their inability to synthesize the differences

among their representations of the topics. Recursive architectonic software would be aimed to support the social development process by being able to identify the order of phase complexity and phase transitions of each proposition offered and argued for by the differing stakeholders, and contextualize it clearly to all participants. In conjunction, relevant information to a discussion would be easily accessible – even automatically recommended by the software to the participants of the discussion. Long, drawn out arguments that have been repeatedly conducted and rehashed unproductively by other individuals and groups could be avoided because where such conversations had already transpired and solutions had been presented, this history would be immediately associated with and brought into the immediate instance of the discussion.

Fourth, the total vacancy of information correspondence, the holes in human knowledge, the uncharted territory of innovation might be more easily identifiable in recursive architectonic software. As the database fills with qualia assemblies, different permutations of qualia assembly across different domains of human knowledge across different orders of phase complexity may be clearly articulated, helping both lay people and academics alike to identify new frontiers of knowledge. Further, the entire lineage of all participants in the global project would be preserved for future generations, giving a wider range of the global population an opportunity to help shape our shared understanding of reality.

Fifth, models, frameworks, and information in any form that contribute to knowledge about any and all aspects of the universe across all scales and complexity, tend to be presented in a static form. This recursive architectonic software would be designed as a dynamic model; it would allow a retaining of original works, but also allow

them to be dynamically improved through global interaction. And so, too, for the recursive architectonic software as a whole; though the software could be used without users making any direct reference to the underpinning architectonic framework or having any care to do so, the architectonic itself would undergo improvement as it integrates the multiplicity of human knowledge, both automatically as well as by the hands of intentional global differentiation and integration across all fields of knowledge. Thus, even the contributions of those whose work is more specific and limited in scope would indirectly help the evolving theory of everything that is the recursive architectonic, along with those who do so intentionally.

Sixth, there are often many stakeholders in any decision-making process, and not all voices of stakeholders are heard or integrated into decision making processes. Another benefit of recursive architectonic software is that it would provide a framework for individuals, groups, institutions, communities, and even states, nations, and internationals to enact global participation in decision making processes. Individuals across the globe would have a means for voicing their perspective on decisions that affect them, framed in a software built specifically for that purpose. This is aimed to empower everyone's voice, no matter their life conditions, beliefs, nationality, complexity, and socioeconomic status.

Seventh, this recursive architectonic is in large part intended to be a starting point for a true theory of everything that includes all domains of human knowledge, and actions for coordinating them. It is my intention to create software that can enable global participation in a dynamic theory of everything that evolves with humans.

Thesis Summarization

Throughout this thesis, I have reviewed the state of philosophy of mathematics, traced the roots of computability theory, and resolved Gödel's theorems to their foundations as natural qualia abstraction and orders and transitions of human behavior – and possibly even to the foundation of the universal properties of the universe that drive evolution forward. I have synthesized together the integral architectonic that is SHIM, and the integral architectonic that is MPC into a recursive architectonic, devised a well-ordered axiomatic organization for their principles, all in a computationally usable format, and synthesized them to their shared properties.

Since the dawn of humankind, humans have sought to better understand themselves and their environment. We live in an age where it is now possible for us as a species to bring together our combined efforts across all domains and methods of knowing, and share in this globally. This thesis is a contribution toward that purpose.

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APPENDIX

Spectrum of Human Imagination Model v84.8, Fractal Hierarchy

The material of this Appendix is an accompanying table.