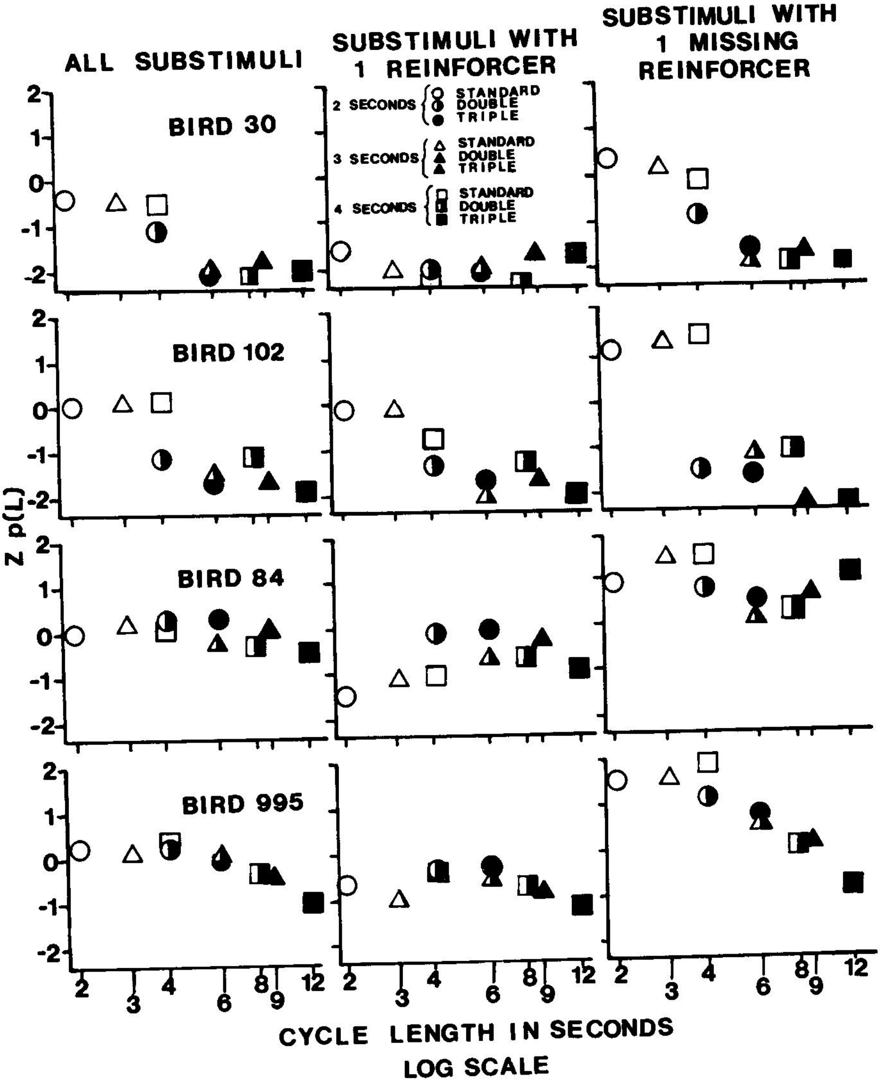
The leftmost panel of Figure 3-4 replots the perceived value (bias) of the stimuli as a function of cycle length. This information was previously shown in the leftmost panel of Figure 3-3, but in Figure 3-4 it is plotted separately from sensitivity. Calling bias, which is produced by a stimulus, perceived density can be justified as follows: If $p(L)$, the tendency to indicate that a sample came from the rich schedule, equals 1, when a sample from the rich schedule has been presented, then the bird has responded correctly and is reinforced. If $p(L)$ equals 0 when a sample from the lean schedule has been presented, again the bird has responded correctly and is reinforced. Hence, the value of $p(L)$ indicates the perceived density of a schedule sample. The tendency to say that the sample came from the rich schedule, $p(L)$, was transformed into the corresponding z value by the probit, or inverse probability, transformation in order to produce more linear functions. The resulting quantity, $z_{p(L)}$, is the perceived density of a schedule sample. The $z_{p(L)}$ was plotted against the log of cycle duration. The first panels show the effect of cycle length at the macro level. The perceived value of a substimulus generally rose or stayed nearly the same as cycle length increased from 2 seconds to 3, either decreased or stayed the same for cycle length from 3 to 4 seconds, and decreased in a not entirely linear fashion for cycle length above 4 seconds.

At the molar level, the perceived value of substimuli with one reinforcer is shown in the middle panels and that of substimuli with three reinforcers in the rightmost panels. While performance characterized at the macro level reflected the birds' scaling of the perceived value of all the substimuli together, the molar level analysis focuses on their responses to substimuli with the same reinforcement densities. The perceived values changed in a reasonable fashion for substimuli with one reinforcer or one missing reinforcer. The panels in the middle and right columns of Figure 3-4 indicate that most of the birds show maximum perceived value at around 4 to 6 seconds. There was no consistent change in perceived value as a function of standard cycle length alone. However, when combined with doubling and tripling on probe trials, there is generally a decrease in perceived value as a function of overall cycle length (i.e., the half-filled and filled points generally fall below the unfilled points). The form of the function is not due to different density substimuli being treated differently, as can be seen from the similarity of patterns in the panels in columns 2 and 3, whereas the height of the function is.

Figure 3–4. Perceived density, $z_{p(L)}$, plotted against cycle length in log seconds. The left-hand column shows the effect of cycle length on the perceived density of all substimuli considered together. This is the macro level relation. At the molar level, Columns 2 and 3 show the relation between perceived density and cycle length for substimuli with one reinforcer and substimuli with three reinforcers (one missing reinforcer).



In the following account of the foregoing performance at both the macro and the molar level, it is predicted that perceived density (the inverse probability transform of the probability of a left-key peck) is a linear function of actual density. First, because of the way that substimuli are distributed between the rich and lean schedules (Commons, 1979), the relative expected payoff, Rel $EP(L|S_{N_d})$, was an ogival function of reinforcement density. The relative expected payoff for a left-key peck at a given density substimulus, $D(S_{N_d}) = N_d$, is just the probability of a left-key peck being correct, LC, at that density, N_d, when amount of reinforcement for each correct choice is equal.

$$\text{Rel } EP(L|S_{N_d}) = \frac{p(LC|S_{N_d})}{p(LC|S_{N_d}) + p(RC|S_{N_d})},$$

$$\text{but } p(RC|S_{N_d}) = 1 - p(LC|S_{N_d})$$

$$= \frac{p(LC|S_{N_d})}{p(LC|S_{N_d}) + (1 - p(LC|S_{N_d})}$$

$$= p(LC|S_{N_d})$$

The counterpart of Rel $EP(L|S_{N_d})$ or $p(LC|S_{N_d})$ in free operant schedules is programmed reinforcement rate. This fact will be used to suggest that the mechanism that gives rise to the matching result will generalize to the more usual free operant schedules.

Second, applying a probit transformation (the inverse of the standard normal cumulative distribution function) to the function that relates relative expected payoff to density turns it into a straight line with slope 1.084, intercept -2.110, and an $r^2 = .996$; $z_{p(LC|S_{N_d})} = -2.1\ (1 - N_d/2)$. This linearity results from the goodness of the normal approximation to the binomial, which was used to generate the stimuli. In Figure 3-5, the independent variable can be expressed in two ways—actual reinforcement density of the substimulus or probability transform of relative expected payoff for indicating that a substimulus had high density. If perceived density is a linear function of actual density, it is also a linear function of relative expected payoff in z form and vice versa. As a consequence, the matching relation

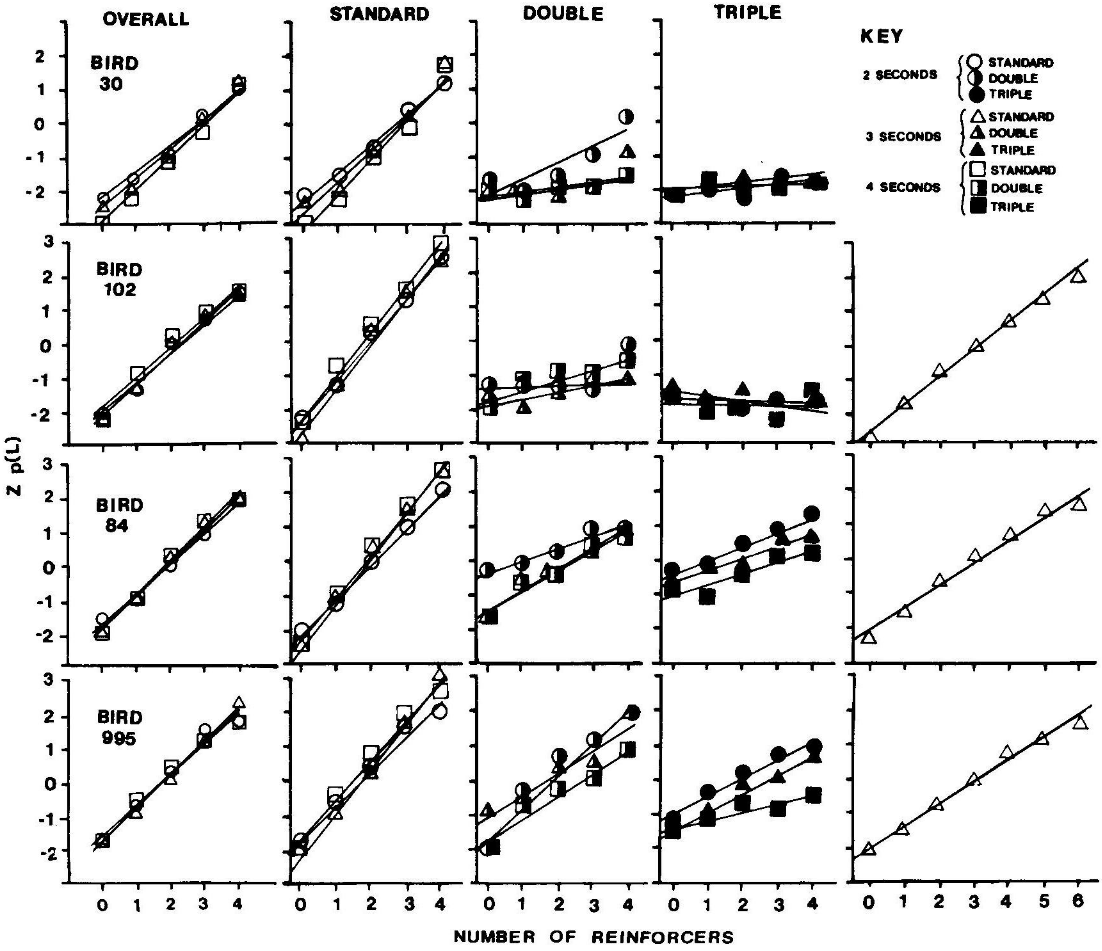


Figure 3–5. Perceived density, $z_{p(L)}$, shown as a function of actual density (the number of reinforcers in a substimulus). This molar relation is shown for all trials together (Column 1), for trials with standard cycle length (Column 2), for probe trials where the standard length was doubled (Column 3) and tripled (Column 4), and for six cycles at standard length (Column 5).

holds (i.e., perceived density is equal to the probit transform of relative expected payoff) if perceived density is found to be a linear function of actual density. Since matching was found to hold for similar substimuli, but without probe trials being included in the sessions (Commons, 1979), it would be predicted to hold here.

Figure 3-5 shows that matching at the molar level holds when the entire session is considered. The relations shown within the figure rejected a number of ways that pigeons might scale reinforcement density in samples, while supporting others. The decision rules for each bird at the molar level are graphed with respect to different parameters. The decision rule examined here is the psychophysical relation between the perceived reinforcement density, $z_{p(L)}$, and the actual reinforcement density—with cycle length, obtained by two different manipulations, being the parameter. The leftmost column of panels in Figure 3-5 shows the molar relation between the perceived density, $z_{\text{average}\, p(L|S_n, D(S_n) = N_d)}$, and actual substimulus density, N_d, for three different cycle lengths for the combination of probe and standard trials. The parameter is cycle length. The points were well described by the regression lines fit by the median method (Mosteller and Tukey, 1977), with the r^2 values ranging from .98 to .99, as shown in Table 3-1.

That the birds linearly scale reinforcement density in the schedule samples of the duration studied here implies that the matching law holds at the molar level, since linear density scaling implies matching, as discussed above. The matching law and linear density scaling were further supported by a number of related facts. In z form, the mean perceived density was 0 for the mean actual density of 2 (density 2 substimuli), as it should have been: This means that these substimuli were seen as coming equally from either distribution. The perceived values were symmetrically distributed about 0 with perceived density ranging from a value of −2.2 for density 0 substimuli to +2.2 for density 4 substimuli.

For the standard cycle lengths alone in Figure 3-5, functions relating perceived density to number of reinforcers were steeper than those for the combined trials. This suggests if matching is fundamental, then the birds compensate for the inclusion of decrement-producing probe trials by overvaluing substimuli on standard trials. The r^2 values for the best fit lines range from .98 to .99.

One can see how density was scaled by examining the role played by two ways of lengthening cycles in producing the decrement in

Table 3–1. Decision Rules at the Molar Level.

		Slope *Z P(L)/Density* *(Z EP(L)/Density)*	*Intercept* *Z P(L)*	R^2	*Ratio of Slopes of Standard Cycle Length to Given Slopes*	*Model I Predicted Ratios of Slopes*	
Predicted by matching		(1.084)	-2.11	.992			
Overall	2 sec	0.919	-1.941	.984	1.464		
	3 sec	0.979	-2.04	.972	1.230		
	4 sec	0.953	-1.917	.975	1.282		
Standard	2 sec	1.054	-2.16	.992	1.000		
	3 sec	1.204	-2.37	.993	1.000	1	
	4 sec	1.222	-2.22	.993	1.000	1	
Double	2 sec	0.485	-1.462	.556	2.17	2	standard/ double
	3 sec	0.421	-1.735	.457	2.86	2	
	4 sec	0.451	-1.741	.659	2.71	2	
Triple	2 sec	0.234	-1.286	.106	4.50	3	standard/ triple
	3 sec	0.226	-1.402	.155	5.34	3	
	4 sec	0.112	-1.557	.087	10.91	3	
Six cycle standard	3 sec	0.690	-2.21	.976			

sensitivity. Again, the two ways were to change cycle length on selected probe trials by doubling or tripling the standard cycle length or to change the standard cycle length over an extended period of time. Any model that suggests that the birds responded simply on the basis of number of reinforcers, independently of context or time, must predict that momentary changes in cycle length should have no effect on perceived value. Any model that proposes that the birds responded on the basis of the relative time between reinforcers or rate of reinforcement must predict that perceived value should be inversely proportional to momentary cycle length. If the standard cycle is doubled on a series of trials, the perceived value should be halved. Likewise, the ratio of the slopes of functions relating perceived density to actual density should be halved. A third alternative is that the birds perceived something like a weighted average rate or weighted average time between reinforcers. This model should include a term for the interaction between standard cycle length and ratio of probe length to standard.

How well these various models are supported by the data is seen in all the panels in columns 3 and 4 of Figure 3-5. The perceived substimulus density decreased more with the insertion of probes than with increases in standard cycle length, as is shown by the flattening of the slopes for doubles and triples. The probes caused even greater decrements as the standard cycle length increased.

Any model that depends on number of reinforcers alone is rejected by the fact that there were changes in slope with increased cycle lengths, indicating that time was indeed important. The standard cycle slopes were 1.0, 1.2, and 1.2 at 2, 3, and 4 seconds, respectively; doubling slopes were .49, .42, and .41 (clearly lower); and the tripling slopes were lower still—.24, .21, and .15 (as shown in Table 3-1).

Doubling and tripling standard cycle lengths decreased the perceived density more than predicted by time- or rate-averaging models. The ratio of the slopes, double to standard and triple to standard, would be 2:1 and 3:1, respectively, if the weighted average rate or weighted average time model were true in its simplest form. As shown in Table 3-1, the ratios of the slopes for the average of the four birds are 2.0, 2.9, and 2.9 for 2-, 3-, and 4-second cycle lengths for doubling and 4.2, 5.7, and 8.0 for tripling. There may be an interaction between standard cycle length and probe value: At least for Birds 84 and 995, tripling the 4-second standard had a larger dec-

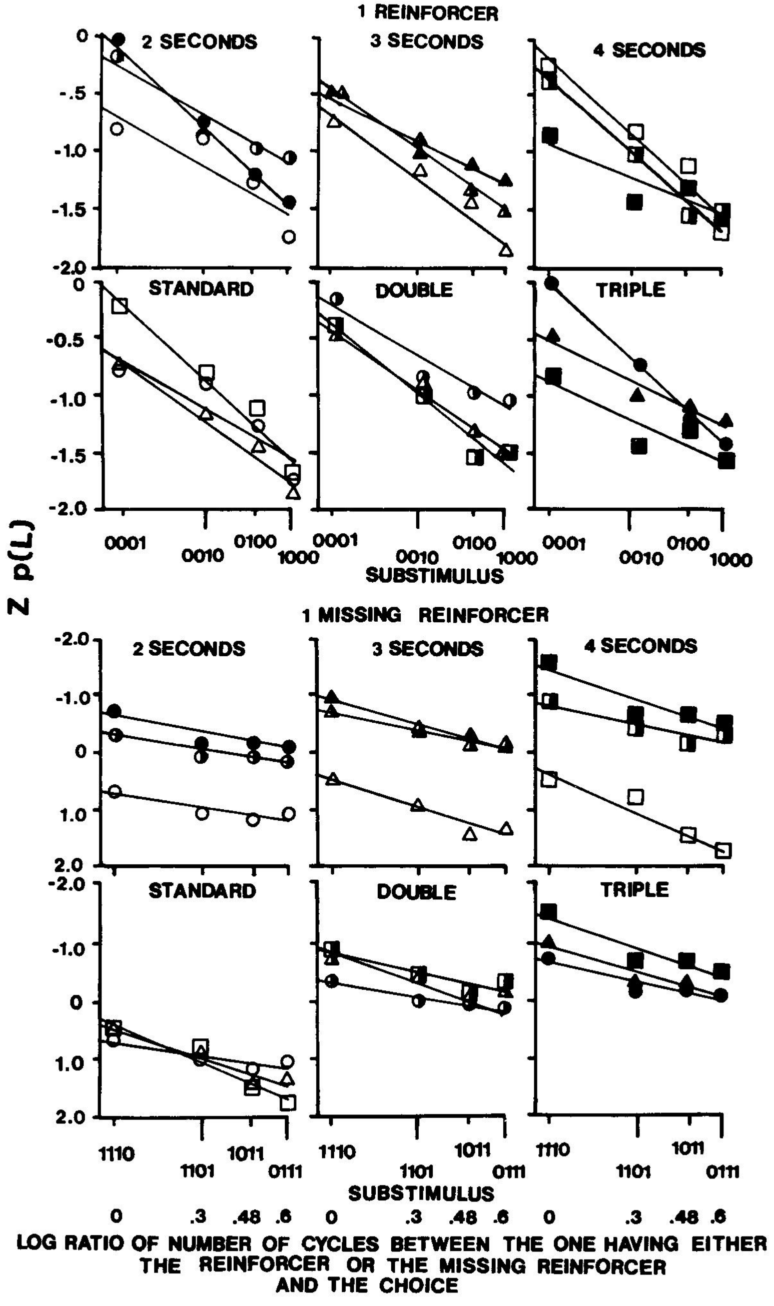
Figure 3–6. Perceived density (or its negative), $z_{p(L)}$ or $-z_{p(L)}$, shown as a function of the log of the ratio of the number of cycles before choice of either the cycle with the single reinforcer or the one missing reinforcers. For substimuli with one reinforcer, the top row shows this as a function of the various cycle lengths. The second row shows this for standard trials and probe trials on which standard cycle lengths were doubled and tripled. The bottom two rows show this relation for substimuli with one missing reinforcer (three reinforcers). The *y* axis is inverted for the bottom two rows so that a direct comparison may be made between the top two rows and the bottom two rows, with row 1 and 3 corresponding and row 2 and 4 corresponding.

remental effect on the slope than tripling the 2-second standard. While these slope changes are in the right direction, they clearly deviate from ratios predicted by time or rate averaging, especially as standard cycle length increases. This is not surprising. The birds do not compensate for the fact that the probe substimuli start much earlier than the standard. The decrement in perceived density is greater than would be the case if those earlier events in the substimuli were not there.

If average reinforcement rate determined perceived density, reinforcers occurring more cycles away from choice would not be weighted less. However, as weights decrease farther from choice, two versions of weighting have to be considered—one that hypothesizes exponential decay (i.e., relative time alone is important) and a second that hypothesizes a limited event window or short-term working memory (i.e., number of intervening events is important).

To see whether this decrease occurs, the effect of a single reinforcer (or single missing reinforcer) occurring farther from choice is examined at the molecular level. Averages of four birds' perceived density are shown in Figure 3-6 as a function of how far the choice is from either a single reinforcement opportunity or a single missing reinforcement opportunity, both here called critical events. Averages of the four birds' performances were used because, at least for the probes that doubled and tripled standard cycle length for each particular substimulus, the frequencies of those substimuli were small, resulting in noise. The data for substimuli with one reinforcer are shown in the top two rows of panels of Figure 3-6, and those for one missing reinforcer in the bottom two rows. A point in Figure 3-6 at density 1 is the average across birds for density 1 substimuli of the same cycle length. Likewise, a point at density 3 is an average

(Figure 3-6)


1 REINFORCER
2 SECONDS
3 SECONDS
4 SECONDS
STANDARD
DOUBLE
TRIPLE
0
-.5
-1.0
-1.5
-2.0
-0.5
0001
0010
0100
1000
SUBSTIMULUS
Z p(L)
1 MISSING REINFORCER
2 SECONDS
3 SECONDS
4 SECONDS
STANDARD
DOUBLE
TRIPLE
-2.0
-1.0
0
1.0
2.0
1110
1101
1011
0111
SUBSTIMULUS
0
.3
.48
.6
LOG RATIO OF NUMBER OF CYCLES BETWEEN THE ONE HAVING EITHER THE REINFORCER OR THE MISSING REINFORCER AND THE CHOICE

across birds of density 3 (one missing reinforcer) substimuli of the same length.

The data are again graphed in two ways. The standard cycle length parameters are graphed as circle (2 seconds), triangle (3 seconds), and square (4 seconds). How much a particular symbol is filled depends on whether it represents a standard (unfilled), doubled (half-filled), or tripled (completely filled) cycle. Some general properties emerge. There is a larger decrement for one reinforcer than for the one missing reinforcer, as shown by the slopes for the former being greater than for the latter. This suggests that the diminution of control is greater for an event rather than for a missing event. Other effects are not as orderly, perhaps because some of the events are occurring very far from choice in time, and $z_{p(L)}$ values are close to either -2 or $+2$.

A Micro-Molecular Model that Accounts for Perceived Density of a Substimulus

I suggest that perceived density of a substimulus, $D^*(S_n) = z^*_{p(L|S_n)}$, is the sum of the weighted values of reinforcers that occurred in the substimulus. On each cycle, c_i, a reinforcer in a substimulus may occur ($v_i = 1$) or not occur ($v_i = 0$). Cycle number, c_i or i alone, is counted backwards from choice and has values 1, 2, 3, 4, the value 1 being right before choice and at the end of the stimulus period.

A multiple regression was performed to determine the contribution of a reinforcer on a given cycle. Remember that a substimulus has four such cycles, so that there are four coefficients, a_i for v_i, and a constant, a_0, to be estimated for this molecular model.

$$D^*(S_n) = z^*_{p(L|S_n)} = a_0 + a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4$$

$$= a_0 + \sum_{i=1}^{4} a_i v_i$$

The weights, a_i, represent the contribution of a reinforcer on a respective cycle, c_i. For substimulus, $S_0 = 0000$, all the v_is are 0, so that a_0 represents the perceived density of a substimulus with no reinforcers. The independent variable was v_i, whether or not a reinforcement opportunity occurred on a given cycle number; the perceived value of each of the sixteen substimuli at 2-second standard cycle length was the dependent variable. As shown in the second column of Figure 3-5, the other functions were almost identical, so only the analysis of the 2-second data is reported. This analysis of all four birds together yielded a molecular model with five estimated parameters, a_0 through a_4, -2.16, 1.40, 1.168, 0.900, and 0.7125, and a multiple r of .91. For individual birds the coefficients were slightly different, and r values were slightly higher ranging from .96 to .99, since variability between individuals was eliminated. Since linear regression depends on the predictor variables being additive, the fact that so much of the variance was accounted for suggests that substimulus density is the sum across the i cycles of the contribution of events on each of the ith cycles. This finding is consistent with Wickelgren's (1974) assumption that the contribution of multiple occurrence of the events to be remembered is additive.

The heart of the matter is to see if Wickelgren's (1974) suggested exponential equation, which represents forgetting, accounts in a more specific fashion for the contribution of the programmed events on a given cycle. While the previous multiple regression reduced the number of parameters necessary to describe the data and showed additivity, using an exponential model should allow a more compact description. The coefficients, a_i, for each cycle were previously the parameters to be estimated. Since the micro model suggested that each term was exponential, the parameters, a_i, now become the dependent variables, and the independent variables, c_i, were used in the expression $a_i e^{b_i c_i}$ to predict them. The values of $a = 1.793$ and $b = -0.229$ so found yielded a correlation between cycle number away from choice and log cycle number weight (from the previous regression) of -0.9975, $r^2 = .995$. Hence, the model's coefficients predicted perceived value as well as the multiple regression and yet had only three parameters for sixteen substimuli. To further test this model of what happens on a given cycle (micromodel), the predicted coefficients were entered into the multiple regression as multipliers of v_i. If the micro model worked well, the multiple r value should be the same as the molecular, in which the coefficients were directly obtained as a best fit ($r = .912$), and the coefficients should be very close to 1. The residuals should not be correlated with cycle number. The obtained r for this procedure was .912; the coefficients were 0.98, 1.03, 1.00, and 0.99; the constant was -0.216; and the residuals were uncorrelated. The combined micro-molecular model equation for the perceived density of a substimulus is:

$$D^*(S_n) = z^*_{p(L|S_n)} = -2.16 + \sum_{i=1}^{4} v_{c_i} \; 1.8e^{-.229c_i}$$

This finding on the form of the decay function is similar to Wickelgren's (1974) in that for the short term, events lose impact as a function of the negative exponential of time, $a_i e^{b_i T_i}$. Some comparisons of the present model for pigeons with his model for adult humans should be made. Here, it was the number of cycles before choice (the relative temporal distance away from choice, c_i) on which each reinforcement and non-reinforcement opportunity fell that accounted for the decay, instead of time away from choice, T_i. The latter had no effect on perceived density, as shown for standard length cycles in column 2 of Figure 3-5. Here, because it was how many cycles before a choice that mattered and not simply time before choice, a pure timing or temporal decay interpretation is unwarranted. Each cycle may act like an event. Knowing how many cycles before choice a given event fell tells us how many intervening events there are. Whether or not the form of the decay function is due to interference of intervening events or simply to the passage of time was not answered by Wickelgren. The fact that number of events to be processed may be the critical variable suggests an examination of the effects of number of cycles on perceived density and sensitivity, the next experiment to be described.

EXPERIMENT II: SIX 3-SECOND CYCLE SAMPLES

Method and Procedure

The subjects and apparatus for this experiment were identical to those described in Experiment I, except that one less subject was used. The discrimination task was similar to that used in Experiment I, except that the number of cycles was increased from four to six. The stimulus on each trial, then was one of 64 substimuli (of six equal duration cycles) randomly selected from one of two T schedules. The two schedules to be discriminated were, again, two