overlapping distributions (stimuli) of reinforcement opportunities—one with $p(S^{R+}|R_C) = 0.767$ (the rich schedule) and one with $p(S^{R+}|R_C) = 0.233$ (the lean schedule). These probabilities were chosen so that the ideal observer's discriminability, d', based upon the overlap of the discrete binomial distribution, was the same as in the four-cycle case. The rich and lean binomial frequency distributions are shown in Figure 3-3.

Results

The d' values reported in this section were found by assuming a continuous binomial model and were then compared to the d' values calculated in the same manner for the same birds in the four-cycle case. These d' values differed slightly from those obtained with the normal approximation to the binomial reported everywhere else in this chapter.

The d' values shown in Table 3-2 were lower for the six 3-second cycle sample discrimination than for the four 3-second cycle sample discrimination, whether the values were found by assuming a continuous binomial or a normal approximation to the binomial. The discrimination efficiency, e, was much lower for the six 3-second cycle sample (18-second long sample) than for the stimuli with four 4-second cycles (16-second long sample). Here, e is the ratio of obtained d' to maximum possible d'. This supports the notion that there is much more of a problem with processing more programmed events than with processing extraneous events occurring over time, such as the house light, which was off, or the hopper light, which was on.

The slopes of the six-cycle decision lines were not as steep as the four-cycle ones. The addition of two extra events to the four-cycle samples decreased sensitivity and altered the decision rules more than might be expected if maximizing one alternative as soon as it became more favorable held instead of matching. If a maximizing account were closer to the truth, the birds might have treated both a density 0 and 1 substimulus the same as a density 0 substimulus and a density 5 and 6 substimulus the same as a density 5 one, thereby maintaining the slope. The fact that they did not shows that their decision rules are not based on a simple strategy such as counting number of reinforcers or discriminating simple rates of reinforcement. Instead, they showed the same pattern of anchoring the end

Table 3–2. A Comparison of Obtained Sensitivity to Maximally Possible Sensitivity.

	Six 3-second Cycles			*Four 3-second Cycles*			*Four 4-second Cycles*		
Bird	*Obtained d'*	*Maximal d'*	*(E)*[a]	*Obtained d'*	*Maximal d'*	*(E)*	*Obtained d'*	*Maximal d'*	*(E)*
102	1.78	2.99	.60	2.0	2.19	.91	2.17	1.19	.99
84	1.83	2.99	.61	1.8	2.19	.82	1.8	2.19	.82
995	1.70	2.99	.57	1.9	2.19	.87	1.7	2.19	.77

a. (*E*) = efficiency.

points of the density distribution to the same extreme perceived values—$p(L) = 0$ for density 0 substimuli and $p(L) = 1$ for density 6 substimuli. These are the same as the perceived values found for the four-cycle case. Also, their performance is necessarily limited by the discriminability of the densities since they discriminated the same densities better in the four-cycle case. They seem to scale density in both cases in a linear fashion, matching perceived density to relative expected payoff for indicating that density.

MATCHING AT THE MACRO AND MOLAR LEVEL IS A CONSEQUENCE OF ADDITIVITY AT THE MOLECULAR LEVEL

In this next section, matching will be shown to be derivable from the present micro-molecular model. While the matching law derived here is for the scaling of reinforcement density, Commons and Ducheny (1981) show that a similar model works for a preference situation that reflects response strength. To show how the matching relationship is determined at the micro-molecular level for the scaled density, the terms in the model are put in a form closely akin to traditional matching language. Here a matching relation is between relative response probability (probability and rate are similar measures) and relative payoff for the response indicating the higher density. Relative response probability is just response probability, since $p(L) + p(R) = 1$, making $p(L)/(p(L) + p(R)) = p(L)$. Relative response probability is then just perceived density. The micro-molecular model showed that each reinforcer makes an equal contribution to relative perceived density when it is exponentially weighted by the decrement in control. The aggregated effect of the total number of reinforcers in a substimulus is the simple sum of the effects of each single reinforcer.

The fact that perceived substimulus density was shown to be the sum of the contributions of each reinforcer on the cycle in which it occurred implies that perceived density is matched to the payoff for indicating the higher density. The proof involves two steps—how molar level substimuli are defined and how linearity implies matching. The contribution of a reinforcer was shown to be an exponentially decaying function of how far the cycle precedes choice.

Perceived reinforcement density at the molar level is for an entire set of substimuli with the same number of reinforcement opportunities. Each reinforcer that contributed to the density of the group was equally likely to fall in any one cycle. Since for each individual substimulus, the effects of a reinforcer on perceived density add, the number of reinforcers occurring in the molar level substimulus class would be additive.

In the derivation of matching at the molar level from the molecular model, the perceived density, $D''(S_{N_d})$, of the molar substimulus, S_{N_d}, is defined in a slightly different manner than previously. Here the perceived density is the average of the perceived density of each same density substimulus, $z_{p\,(L|S_n)}$, already in z form, rather than the z transform of the average $p\,(L)$s. The matching relation for this perceived value and payoff for the 2-second standard data is just as good; the r^2s were all greater than .98 and equal two places beyond the decimal point.

Some of the properties that make this derivation possible depend on empirical properties embedded in the molecular model—specifically, the symmetrical dispersion of $D^*(0000)$ and $D^*(1111)$ around 0. This allows the value of the Σa_i, the sum of the contributions of each reinforcer, to be found in terms of a_0, the perceived value of (0000).

The following molecular model is assumed to be true:

$$D^*(S_n) = z^*_{p\,(L|S_n)} = a_0 + \sum_{i=1}^{4} v_i a_i$$

Because the value of the contributions of a reinforcer on a given cycle, Σa_i, will be needed later, it is found next.

$$\text{The value } D^*(0000) = a_0 + \sum_{i=1}^{4} 0 \cdot a_i = a_0 = -2.16$$

$$\text{and the value } D^*(1111) = a_0 + \sum_{i=1}^{4} 1 \cdot a_i = a_0 + \sum_{i=1}^{4} a_i$$

$$= 2.03 \approx 2.16$$

Then, by solving the equations simultaneously:

$$\sum_{i=1}^{4} a_i = -2a_0$$

The next part of the derivation shows how many a_is appear in each molar substimulus, S_{N_d}. It is necessary to know how many substimuli of the same density there are in each molar substimulus. The cardinality of S_{N_d} is just the combinations of 4 cycles taken N_d at a time:

$$CS_{N_d} = C\left\{S_n \mid D(S_n) = N_d\right\} = \binom{4}{N_d} = \frac{4!}{N_d!\,(4-N_d)!} \quad \text{Then}$$

$$D^*(S_{N_d}) = \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} D(S_n)$$

$$= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} \left(a_0 + \sum_{i=1}^{4} a_i v_i\right)$$

To expand this expression, the appropriate number of 1s and 0s have to be put in for v_i for each of the substimuli of a given density. The number of reinforcers ($v_i = 1$) per substimulus is N_d; the number of distinct substimuli, S_n, in the class of same density substimuli, S_{N_d}, is $\binom{4}{N_d}$. Therefore, a total of $N_d\binom{4}{N_d}$ terms with $v_i = 1$ occur in the sum. Each of the four cycles contains a reinforcer for an equal fraction of the time, so the total number of times each a_i is multiplied by a $v_i = 1$ is $\frac{1}{4}N_d\binom{4}{N_d}$. Hence:

$$D^*(S_{N_d}) = \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} \left(a_0 + \sum_{i=1}^{4} a_i v_i\right)$$

$$= \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} a_0 + \frac{1}{\binom{4}{N_d}} \sum_{S_n \in S_{N_d}} \sum_{i=1}^{4} a_i v_i$$

$$= \frac{\binom{4}{N_d}}{\binom{4}{N_d}} a_0 + \frac{N_d}{4} \frac{\binom{4}{N_d}}{\binom{4}{N_d}} \sum_{i=1}^{4} a_i$$

$$= a_0 + \frac{N_d}{4} \sum_{i=1}^{4} a_i$$

$$\text{but} \sum_{i=1}^{4} a_i \approx -2a_0$$

Therefore,

$$= a_0 - \frac{N_d}{2} a_0 = a_0\left(1 - \frac{N_d}{2}\right) = D^* (S_{N_d})$$

This last expression is the same for the relative payoff probability shown earlier, so the derivation is complete.

WHY HIGHER LEVEL MODELS FAIL

Schedule samples have a number of characteristics. At the macro level, they may differ in overall density. While much may be learned from an examination of the relationship between gross density and response rate or choice, what is controlling about schedules and how that control is exerted may lie at a number of lower levels. First, one would expect the most orderliness at the level that the contingencies constrain the most and at the level where the mechanisms embedded within the contingencies work. Here this took place at the molecular and micro levels. The macro level performance simply reflects the accumulation of all the values from the micro, molecular, and molar levels.

There have been three demonstrations that, in a density discrimination situation, birds match their choices to relative expected payoff for making these choices. In the first (Commons, 1979), relative expected payoff was shown to encompass the effects produced by varying the amount of reinforcement for a correct discrimination of density. Matching was obtained to relative expected payoff for an aggregation of standard and probe trials, with changes in perceived value at the standard lengths compensating for the changes at probe lengths. Matching was also obtained when the number of cycles was increased with the discriminability between schedules held close to constant.

There are a number of implications that follow from matching having been obtained. For the small range investigated here of 0 to 4 and 0 to 6 reinforcers, perceived density was linearly related to actual density, with increments in one producing proportional increments in the other. The perceived value is reflected by probabilities ranging from very close to 0 ($z = -2.2$) to close to one ($z = +2.2$). The $z_{p(L)}$ values representing perceived density are almost perfectly correlated with the actual densities ranging from 0 to 4 and 0 to 6. This suggests that perceived density lies on a ratio scale. However, the fact that perceived density is linearly related to actual density at the molar level indicates very little about the perceptual process for two reasons:

1. The effect of changing cycle length does not produce a simple change in perceived density; and
2. The perceived value of different substimuli with the same average density varies a great deal.

One class of molar model states that perceived density simply reflects the total number of reinforcers obtained on a trial, irrespective of when they occurred or their distribution. That momentary increases in cycle length produce decrements in perceived value leads to a rejection of these models, because birds should have been insensitive to the temporal spacing.

Any molar model that states that perceived density reflects average reinforcement rate or average interreinforcement time was also rejected as an overall model. These models were closer to the truth, especially if an interaction of momentary increases in cycle length with standard cycle length is allowed. One reason they were rejected is summarized in Table 3-1, where it is shown that the ratio of slopes of the standard to the obtained perceived value greatly exceeded the predicted value of 2 and 3 for cycle length doubling and tripling, respectively. A second reason for rejecting these molar models was that the contribution of a reinforcer to perceived density decreased as a function of how far before choice that reinforcer occurred, as was also found by Commons (1979). Here, when the probability measure of perceived value was put in z form and number of cycles away from choice of the critical event was logged, the relation between the two was approximately linear, although there was a good deal of variability for some individual points. A slightly better fit would have been obtained if the exponential transformation used in the micro-molecular model had been used.

To the extent that the micro-molecular model is successful, it suggests the following processes: Perceived density is determined by decremental weighting, with the number of reinforcers weighted by a negative exponential of their number of cycles before choice. Changes in cycle length did not appear to change the exponent. The effects of the number of events or expected number of events or expected duration of a stimulus period may potentially be described by the coefficient multiplying the exponential terms. Narens (1979) has suggested that varying cycle length from cycle to cycle within a trial, while keeping overall substimulus length constant, may clarify some of these interference and timing issues. Herrnstein (1980) has suggested using fractional probes such as one-half the standard cycle length to see the effect of the probe per se. Including probes that change cycle number but not overall substimulus duration should also be tried.

The results here are entirely consistent with those found by Mandell (Chapter 2). Changing standard cycle length produced minimal changes in discriminability, and the relative time between reinforcers within a substimulus was much more important in determining performance than absolute cycle length in degrading performance. One would predict that discrimination of much leaner densities—that is, much greater cycle lengths—would not be terribly difficult.

SUMMARY

The way that reinforcers sampled from a schedule are discriminated and scaled is best explained at the micro and molecular level. Each reinforcer as it is weighted makes an equal additive contribution to both the discriminative choice behavior as measured by d' and the scaled or perceived density as measured by $z_{p(L)}$. The weighted value of a reinforcer was the negative exponential of how many cycles before choice that reinforcer fell. Since cycle length per se did not matter, a decay theory based on number of intervening events was supported. Sensitivity was lower when six 3-second cycle samples were discriminated than when the equivalent number of 4-second cycle samples were discriminated, also supporting a decay theory based on number of intervening events. Furthermore, probes that momentarily increased cycle length (and hence sample length) caused a decrease in perceived density greater than the amount predicted by a model asserting that rate or time between reinforcers was being scaled. This also raises the possibility that, to the bird, the momentarily increased sample length was perceived as an increase in the number of events to be processed, some of which may have fallen outside of the time or number of events window that the bird is prepared for. This supported a notion that number of events embedded in some time base was controlling perceived density and discriminability.

At the molecular level, the perceived substimulus density is simply the sum of the weighted values of the reinforcers occurring within the sample substimulus. This value systematically deviates from matching, with the reinforcers occurring near choice being overscaled and the ones far way being underscaled. However, at the molar level, matching has been shown to be a direct consequence of the micro-molecular level model holding. The scaled density is not only proportional to the actual density but is equal to the probability of reinforcement for indicating that the particular molar substimulus was rich when put into *z* form. This in turn implies that at the macro level, the scaled value of the overall schedules was the same as the perceived value of the molar components with density 3 and density 1. The rich stimulus had a mean density of 3 and had a corresponding scaled density equal to the molar density 3 substimulus. Likewise, the lean substimulus had a mean density of 1 and a scaled density equal to a density 1 substimulus. The discriminability degradations were also completely accounted for by the micro level performance. The weight of each reinforcer decreased as the reinforcer occurred further and further before choice. This decrement accounted for the decrease in d' from the maximum value of an ideal observer. The details of the latter statement are left to a further exposition.

In Commons and Ducheny (1981), it will be shown that a micro-molecular model in a preference situation (where the substimulus effectiveness rather than scaled value is established) will also explain most of the variance. Since contributions of reinforcers on a cycle will also be additive, the matching law relating the strength of responding to the relative amount of reinforcement will also be demonstrated. What is yet to be done is to show how general the micro-molecular model is, as the length and number of cycles are increased to the values usually found in VI schedules.

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