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INTRODUCTION TO THE MODEL OF HIERARCHICAL COMPLEXITY AND ITS RELATIONSHIP TO POSTFORMAL ACTION

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The Model of Hierarchical Complexity is introduced in terms of its main concepts, background, and applications. As a general, quantitative behavioral developmental theory, the Model enables examination of universal patterns of evolution and development. Behavioral tasks are definable and their organization of information in increasingly greater hierarchical, or vertical, complexity is measurable. Fifteen orders of hierarchical complexity account for task performances across domains, ranging from those of machines to creative geniuses. The four most complex orders are demonstrated by postformal stages of thought, which measure beyond *formal operations,* the highest stage found by Piaget for adults.

KEYWORDS: Behavioral, development, evolution, information, Model of Hierarchical Complexity, performance, postformal, stages, tasks.

This introduction to the Model of Hierarchical Complexity describes its basic terms and concepts, presents the orders of complexity and transition steps between stages of performance, introduces applications of the Model, and lays out its historical origins. The last 4 of the 15 stages of hierarchical complexity are referred to as *postformal thought,* highlighted in this special issue. Thought *is* action. These stages are briefly introduced here, and other articles in this issue will bring those and many other stages to life.

The Model of Hierarchical Complexity (MHC) (Commons, Trudeau, Stein, Richards, and Krause, 1998; Commons and Richards, 1984a, 1984b) offers a standard method of examining the universal patterns of evolution and development. It is a quantitative behavioral developmental theory (the formal theory is presented separately in this issue). There are two kinds of hierarchical complexity. The commonly recognized one refers to the ubiquitous linear hierarchies that are described in many fields of study. These are descriptive. By contrast, the Model of Hierarchical Complexity offers a standard method of examining the nonlinear activity of constructing the universal patterns of evolution and development. It accounts for evolution and development by recognizing their patterns are *comprised of tasks*,

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or actions, *performed* at specified *orders of* hierarchical complexity. Whereas the Model's unidimensional *measure* is linear, the *tasks* it measures are *nonlinear performances*, as this special issue conveys. The nonlinear activity of tasks is that of organizing, or coordinating, information. Hierarchical complexity applies to any events or occasions in which information is organized. The kinds of entities that organize information include humans and their biological systems as well as their social organizations, non-human organisms, and machines, including computers.

The reason it applies so broadly is that it is a singular mathematical method of measuring tasks, and the tasks can contain any kind of information. Thus, its use of purely quantitative principles makes it universally applicable in any context. While it eliminates dependence on mentalistic, cultural, or other contextual explanations, concepts in the Model that address such influences are introduced in other articles in this issue.

As a quantitative behavioral developmental theory, the Model includes a validated scoring system (see Dawson-Tunik, 2006, for hierarchical complexity validation studies). Through seven studies to date, Dawson-Tunik's (2006) work has validated the consistency with which hierarchical complexity theory accounts for stages of development across multiple other instruments that were designed to score development in specific domains. Along with other studies she performed, these support the claim that "the hierarchical complexity scoring system assesses a unidimensional developmental trait" and thus "satisfies the first requirement for good measurement, the identification of a unidimensional, context-independent trait" (p. 445). This enables a standard quantitative analysis of complexity in any setting, a developmental metric applicable to diverse scales that eliminates dependence on mentalistic or contextual explanations (e.g., mental schema, culture). The MHC does not dismiss the influences of other environmental variables on tasks performance. It simply does not quantify those other variables in the measurement process.

TERMINOLOGY

Four basic terms are essential in discussing the Model: *orders, tasks, stage,* and *performance*. The orders are the ideal forms prescribed by the theory's axioms. They are the constructs used to refer to the Model's levels of complexity. The orders of hierarchical complexity are objective because they are grounded in the hierarchical complexity criteria of mathematical models (Coombs, Dawes, and Tversky, 1970) and information science (Commons and Richards, 1984a, 1984b; Commons and Rodriguez, 1990, 1993; Lindsay and Norman, 1977). Tasks are quantal in nature. They are either completed correctly—and thus meet the definition of task—or not completed at all. There is no intermediate state. An example is the adding of two numbers: it can be done only correctly or not at all. Tasks differ in their degree of complexity. The MHC measures the performance of tasks in terms of distinct stages, and it characterizes all stages as distinct. The term *stage* is used to refer to an actual task performed at an order of hierarchical complexity: order is the ideal form, stage is the performed form. Performance is understood as the organization of information. Performance, like the tasks themselves, is *quantal* in nature. That is, there are no intermediate performances. Tasks are understood as the activity of organizing information. Each task's difficulty has an order of hierarchical complexity required to complete it correctly. For example, the task of adding numbers correctly is the necessary condition before performing the task of multiplying numbers. The successful completion of the tasks of adding and of multiplying are examples of two different stages of performance that can be quantified using the MHC. These stages vary only in their degrees of hierarchical complexity. This objective, quantal feature of tasks and stages means that discrete ordinal scores can be assigned to them.

Example to apply the concepts of task, performance, and stage: Organizations' human resource departments may have a list of discrete job responsibilities that are specified for each employment position. Each responsibility represents a task. They may screen potential employees by their ability to perform the tasks required in a particular job. If the screening test used the MHC scoring system, then each task would be assigned an ordinal score. If an applicant tested successfully as being able to perform a specific task, then the applicant's stage of performance on *that* task would match the task's score, and the applicant would be suited for that part of the job. If an applicant was unsuccessful in performing a tested task, then his or her stage of performance would be lower than that task's score, and indicate an area of special attention for the human resource personnel.

BRIEF HISTORY OF STAGE

Ever since the introduction of the idea that development proceeds in discrete stages (Baldwin, 1895; Rousseau, 1979), many models were presented to conceptualize development, including the mentalistic theory of Jean Piaget (1954), a pioneer in the field of developmental psychology. Although Piaget's theory did not define all stages precisely, it clearly established that there is one invariant pathway along which stage development proceeds irrespective of content or culture (e.g., Piaget, 1976). Other developmental models followed Piaget's, and each usually focused on development within a particular domain of information. As more content-oriented models were introduced, the "theme of uniqueness [of each model] was increasingly dropping out" (Kohlberg, 1990, p. 264). Because the varying informational frameworks of different domains have often concealed the common underlying process of stage development, standardization of research methods has been difficult to achieve. Nevertheless, researchers soon recognized the need for a broadly applicable model of developmental assessment that is necessary in order not only to better conceptualize the patterns and themes of development, but also to conduct comparable cross-cultural studies.

THE MODEL OF HIERARCHICAL COMPLEXITY

This section is a summary view of the Model of Hierarchical Complexity to draw together some of the concepts that have been introduced earlier, complete the introduction by defining its term, *hierarchical,* and present the series of orders and transition steps. The MHC classifies the task-required hierarchical organization of actions. Every task contains a multitude of subtasks (Overton, 1990). When the subtasks are completed in a required order, they complete the task successfully. Tasks vary in complexity in two ways, which are defined next: they are either *horizontal* (involving classical information) or they are *vertical* (involving hierarchical organization of information).

*Horizontal (*Classical Information*) Complexity*

Classical information theory (Shannon and Weaver, 1949) describes the number of "yes–no" questions it takes to do a task. For example, if one asked a person across the room whether one penny came up heads when they flipped it, their saying "heads" would transmit 1 bit of "horizontal" information. If there were two pennies, one would have to ask at least two questions, one about each penny. Hence, each additional one-bit question would add another bit. Horizontal complexity is built by the accumulation of bits of information about any event. For example, people could have a four-faced top with the faces numbered 1, 2, 3, or 4. Instead of spinning it like a top, they could toss it against a backboard as one does with dice in a game. For a person outside the room to find out which number appeared in the topmost face of the top after it landed, information-accumulation would require two bits. One could ask whether the face showed an even number. If it did, one could then ask if it were a 2. The possible answers would be either "yes" it was a 2 or "no." If the answer were "no" then by deduction one would know that the topmost face showed a 4. It required only two bits of information to find out which face showed on the top without seeing it firsthand. *Horizontal complexity*, then, is the sum of bits required by just such tasks as this. The tasks involve organizing information that is gathered cumulatively, that is, horizontally.

*Vertical (*Hierarchical*) Complexity*

By contrast, when the task requires the organization of information in the form of action in two or more subtasks, we say this is *vertical* complexity. Hierarchical (vertical) complexity refers to tasks that require the performance of lower-level subtasks before, and in order to, perform more complex tasks. Another way to say this is that less complex task actions are organized, that is, coordinated, by more complex ones. The arithmetic example illustrates this. The ability to add numbers is the lower level task required before one can perform multiplication.

The hierarchical complexity of tasks, or actions, is defined in words as follows. Actions at a *higher order of hierarchical complexity*: (a) are themselves *defined* in terms of actions at the *next lower* order of hierarchical complexity; (b) *organize* and *transform* the lower-order actions; (c) produce organizations of lower-order actions that are new and *not arbitrary.* These next higher order actions cannot be accomplished by those lower-order actions alone. Once these conditions have been met, we say the higher-order action *coordinates* the actions of the next lower order. Thus, *hierarchical complexity* refers to the number of recursions that the coordinating actions must perform on a set of primary elements. Recursions are involved in every hierarchically complex task, from the arithmetic example (where addition is the lower-order action that is coordinated a certain way to perform multiplication) to an accurate analysis of why terrorism exists. Such an analysis requires that many more lower orders of complexity be recursively coordinated before it can be performed. It is *vertically* more complex than multiplication.

Combinations of Lower-Order Actions

Because the Model of Hierarchical Complexity proposes that stage change consists of combining old actions into new ones, it is important to discuss the number of different kinds of combinations of lower-order actions that can occur. There are *iterations*, *mixtures*, *chains*, and *new-stage behavior*. *Iteration* is doing the same action over and over. For example, adding $1 + 2 + 3 + 1 + 2 + 3$ is an iteration of adding. *Mixtures* of actions may include doing a problem set containing simple addition and simple multiplication tasks. *Chains* involve the ordering of subtask actions, but have an arbitrary order. For example, someone could wake in the morning and start the coffee brewing, then do an exercise regimen. The order is arbitrary, because the order could be reversed, for example, the exercise regimen could be done before starting the coffee brewing. According to the Model, when tasks are combined in a nonarbitrary order, then they are *coordinated.* When two or more lower-stage tasks are thus coordinated, there is *new-stage behavior.*

Figure 1 illustrates the pattern of vertical complexity of new-stage behavior from lower to higher orders that applies regardless of the content or context of the tasks. Abbreviated to only six orders to accommodate space limitations, it indicates that each higher-order task coordinates at least two actions at the preceding order's level of complexity. As an illustration of the structure of the ordinal-based system, the graphic's proportions are not intended to represent Log_2scaling.

Tasks

One major basis for this developmental theory is task analysis. The study of ideal tasks, including their instantiation in the real world, has been the basis of the branch of science that studies stimulus control, Psychophysics. Tasks are defined as sequences of contingencies, each presenting stimuli and requiring a behavior or a sequence of behaviors that must occur in some non-arbitrary fashion. Properties

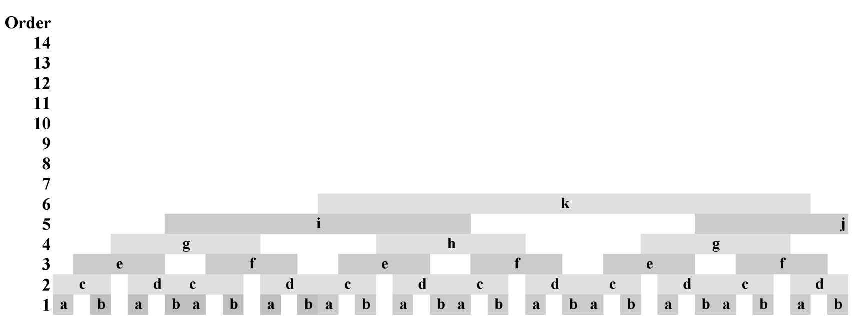

Figure 1. Representation of orders' hierarchical coordination of lower-order actions. Copyright © 2007–2008 by Sara N. Ross. Reproduced with permission.

of tasks (usually the stimuli) are varied and responses to them are measured and analyzed. In the present use of task analysis, the complexity of behaviors necessary to complete a task can be specified using the complexity definitions described next. One examines stage of task performance, that is, behaviors, with respect to the analytically known hierarchical complexity of the task.

Stage of Performance

Stage of performance is defined as the highest-order hierarchical complexity of the task performed or solved. This is why the terms *stage* and *order* should not be used interchangeably, although they sometimes are. The hierarchical complexity of a given task predicts stage of a performance if that task is completed correctly (Commons, Goodheart, and Dawson, 1997; Commons, Richards, Trudeau, Goodheart, and Dawson, 1997). This enables clear distinction between task and the stage of performance of the task. These are two separate concepts that are essential in this Model. The fifteen orders of hierarchical complexity are listed in Table 1. Because these apply to any scale, the orders are fractal (for more discussion of the Model's fractal characteristics, see "Fractal Transition Steps to Fractal Stages: The Dynamics of Evolution, II," this issue).

The Transition Steps

Commons and Richards (2002) discuss the requirements of a robust developmental theory and reviewed other developmental behavior theories. They state that a developmental theory "should account for three aspects of behavior: (a) what behaviors develop and in what order, (b) with what speed, and (c) how and why development takes place" (p. 159). Both simple and complex behaviors should be addressed if a theory is robust. In addition to the stages of development, their transition steps address how and why development takes place, and shed light on factors that affect the speed of development.

They systematized the transition steps originally described in the Piagetian tradition and added a step and substeps based on choice theory and signal detection (Richards and Commons, 1990, as cited in Commons and Richards, 2002), showing how transition steps involved alternations of previous stage tasks. As transition continued, the alternations increased in rate, until the tasks were "smashed" together. At whatever point these were eventually coordinated, behavior at the next stage was formed. The transition step sequence, including substeps, is shown in Table 2.

HISTORY

The following is adapted from Brown (2004). It is important that any "stage" theory and the accompanying scoring scheme have a mathematically and logically developed basis. The pre-Socratic Greek philosopher and scientist, Thales of Miletus (640–546 BC), who had knowledge of Egyptian geometry and Babylonian astronomy, is credited with founding mathematics as a deductive science,

Table 1
Orders of Hierarchical Complexity and Structures of Tasks

Order Ordinal and Name	General Descriptions of Tasks Performed
0 Calculatory	Exact without generalization. Task: simple machine arithmetic on 0s, 1s
1 Sensory or motor	Discriminate in a rote fashion, stimuli generalization, move; move limbs, lips, eyes, head; view objects and movement. Discriminative and conditioned stimuli. Task: Either see circles, squares, etc., or instead, touch them. ◯ □
2 Circular sensory-motor	Form open-ended classes; reach, touch, grab, shake objects, babble; Open ended classes, phonemes. Task: Reach and grasp a circle or square. ◯ □
3 Sensory-motor	Form concepts; respond to stimuli in a class successfully. Morphemes, concepts. Task: A class of open squares may be formed □ □ □ □ □
4 Nominal	Find relations among concepts. Use names; use names and other words as successful commands. Single words may be ejaculatory and exclamatory, and include verbs, nouns, numbers' names, letters' names. Task: That class may be named, "Squares."
5 Sentential	Imitate and acquire sequences; follow short sequential acts; generalize match-dependent task actions; chain words together. Use pronouns. Task: The numbers, 1, 2, 3, 4, 5 may be said in order.
6 Pre-operational	Make simple deductions; follow lists of sequential acts; tell stories. Count random events and objects; combine numbers and simple propositions. Use connectives: as, when, then, why, before; products of simple operations. Task: The objects in a row of 5 may be counted; last count called 5, five, cinco, etc. * * * * * □ □ □ □ □ ◯ ◯ ◯ ◯ ◯ □ /"} Q
7 Primary	Simple logical deduction and empirical rules involving time sequence. Simple arithmetic. Can add, subtract, multiply, divide, count, prove, do series of tasks on own. Times, places, counts acts, actors, arithmetic outcome from calculation. Task: There are behaviors that act on such classes that we call simple arithmetic operations. $1 + 3 = 4$; $5 + 15 = 20$; $5(4) = 20$; $5(3) = 15$
8 Concrete	Carry out full arithmetic, form cliques, plan deals. Do long division, follow complex social rules, take and coordinate perspective of other and self. Use variables of interrelations, social events, what happened among others, reasonable deals. Task: There are behaviors that order the simple arithmetic behaviors when multiplying a sum by a number. Such distributive behaviors require the simple arithmetic behavior as a prerequisite, not just a precursor. $5(1 + 3) = 5(1) + 5(3) = 5 + 15 = 20$
9 Abstract	Discriminate variables such as stereotypes; use logical quantification; form variables out of finite classes based on an abstract feature. Make and quantify propositions; use variable time, place, act, actor, state, type; uses quantifiers (all, none, some); make categorical assertions (e.g., "We all die."). Task: All the forms of five in the five rows in the example are equivalent in value, $x = 5$.

(Continued on next page)

Table 1
Orders of Hierarchical Complexity and Structures of Tasks *(Continued)*

10 Formal	Argue using empirical or logical evidence; logic is linear, one-dimensional; use Boolean logic's connectives (not, and, or, if, if and only if); solve problems with one unknown using algebra, logic, and empiricism; form relationships out of variables; use terms such as if . . . then, thus, therefore, because; favor correct scientific solutions. Task: The general left hand distributive relation is $x * (y + z) = (x * y) + (x * z)$
11 Systematic	Construct multivariate systems and matrices, coordinate more than one variable as input; situate events and ideas in a larger context, that is, considers relationships in contexts; form or conceive systems out of relations: legal, societal, corporate, economic, national. Task: The right hand distribution law is not true for numbers but is true for proportions and sets. $x + (y * z) = (x * y) + (x * z)$; $x \cup (y \cap z) = (x \cap y) \cup (x \cap z)$ *Symbols:* $\cup$ = union (total elements); $\cap$ = intersection (elements in common)
12 Metasystematic	Integrate systems to construct multisystems or metasystems out of disparate systems; compare systems and perspectives in a systematic way (across multiple domains); reflect on systems, that is, is metalogical, meta-analytic; name properties of systems (e.g., homomorphic, isomorphic, complete, consistent, commensurable). Task: The system of propositional logic and elementary set theory are isomorphic. x & (y or z) = (x & y) or (x & z) Logic; $x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$ Sets T(False) $\Leftrightarrow$ ϕ Empty set; T(True) $\Leftrightarrow$ Ω Universal set *Symbols:* & = and; $\Leftrightarrow$ = is equivalent to; T = Transformation of
13 Paradigmatic	Discriminate how to fit, and fit, metasystems together to form new paradigms. Includes ability to show that there are no ways to fit together any set of metasystems. $\Omega_1 \circ \Omega_2 = \Psi^a$ *Symbols:* Ω_n = e.g., Algebraic Metasystems; Ω_n = e.g., Geometric Metasystems; Ψ^a = Analytic Geometry as a paradigm
14 Cross-paradigmatic	Fit paradigms together to form new fields. Only by crossing paradigms can the new fields be conceived and formed; it requires the coordination of multiple paradigms to form genuinely new fields.

Note: From "Applying the Model of Hierarchical Complexity" (p. 65), by Commons, Rodriguez, Miller, Ross, LoCicero, Goodheart, and Danaher-Gilpin, 2007. Cambridge, MA: Dare Association, Inc.

that is, organizing mathematics around demonstrating by logical arguments the correctness of one's assertions and calculations.

But if one does not understand the difference between the ideal and the real one can get into trouble. The failure of the Pythagorean school rested with its need to make its assertions absolute. How could one conduct science or have knowledge in general without the possibility that this knowledge corresponds with reality? Later, Plato handled this problem by rejecting the correspondence account of truth.

Table 2
Transition Steps in the Model of Hierarchical Complexity

Step	Relation	Name	Dynamics within Step
		The first three steps are *deconstructive dialectics.*	
1	$a = a'$ with b'	Temporary equilibrium point (thesis)	While still operating with previous stage synthesis, it does not solve all tasks. Deconstruction begins, an extinction process.
2	b	Negation or complementation (antithesis)	Negation or complementation, Inversion, or alternate thesis. Forms a second synthesis of previous stage actions.
3	a or b	Relativism (alternation of thesis and antithesis)	Relativism. Alternates between thesis and antithesis. The schemes coexist, but there is no coordination of them.
		The remainder of the steps are *constructive dynamics.*	
4	a and b	Smash_0 (begins synthesis)	Begins extinction of the limitations of relativism's theses
5		Smash_1 Random hits, false alarms, and misses, low correct rejections	Elements from a and b are included in a nonsystematic, uncoordinated manner. Incorporates various subsets of all the possible elements.
6		Smash_2 More hits, excess false alarms, low misses and correct rejections	Incorporates subsets producing hits at stage n. Basis for exclusion not sharp. Over generalization
7		Smash_3 Correct rejections and excess misses, low hits and false alarms	Incorporates subsets that produce correct rejections at stage n. Produces misses. Basis for inclusion not sharp. Under generalization.
8	a with b	New temporary equilibrium (synthesis and new thesis)	Arrives at a new, temporary equilibrium where all elements are coordinated and "settled."

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We cannot ever know the truth in its complete and pure form. Anything we can say about reality is only a likely story of the ideal truth.

Here, the ideal truth is the mathematical forms of Platonic ideal. An essential element of science is direct observation and interaction with the world. But Plato set forth a very different doctrine, to the effect that knowledge cannot be derived from the senses; real knowledge only has to do with concepts. The senses can only

Table 3
Examples of tasks studied using the Model of Hierarchical Complexity or Fischer's Skill Theory (1980)

Algebra (Commons, in preparation)	Language stages (Commons et al., 2007)
	Leadership before and after crises (Oliver, 2004)
Animal stages (Commons and Miller, 2004)	Loevinger's Sentence Completion task (Cook-Greuter, 1990)
Atheism (Commons-Miller, 2005)	Moral Judgment (Armon and Dawson, 1997; Dawson, 2000)
Attachment and Loss (Commons, 1991; Miller and Lee, 2000)	Music (Beethoven) (Funk, 1989)
Balance beam and pendulum (Commons, Goodheart, and Bresette, 1995; Commons, Pekker, et al., 2007)	Orienteering (Commons, in preparation)
Contingencies of reinforcement (Commons, in preparation)	Physics tasks (Inhelder and Piaget, 1958)
	Political development (Sonnert and Commons, 1994
Counselor stages (Lovell, 2002)	Relationships (Armon, 1984a, 1984b)
Empathy of Hominids (Commons and Wolfsont, 2002)	Report patient's prior crimes (Commons, Lee, Gutheil, et al., 1995)
Epistemology (Kitchener and King, 1990; Kitchener and Fischer, 1990)	Social perspective-taking (Commons and Rodriguez, 1990, 1993)
Evaluative reasoning (Dawson, 2000)	Spirituality (Miller and Cook-Greuter, 1994, 2000)
Four Story problem (Commons, Richards, and Kuhn, 1982; Kallio and Helkama, 1991)	Tool Making of Hominids (Commons and Miller, 2002)
Good Education (Dawson-Tunik, 2004)	Views of the "good life" (Armon, 1984c; Danaher, 1993; Dawson, 2000; Lam, 1995)
Good Interpersonal (Armon, 1989)	
Good Work (Armon, 1993)	
Honesty and Kindness (Lamborn, Fischer, and Pipp, 1994)	Workplace culture (Commons, Krause, Fayer, and Meaney, 1993)
Informed consent (Commons and Rodriguez, 1990, 1993; Commons, Goodheart, Rodriguez, and Gutheil, 2006; Commons, Rodriguez, Adams, Goodheart, Gutheil, and Cyr, 2007).	Workplace organization (Bowman, 1996a, 1996b)
	Writing (Commons and DeVos, 1985)

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deceive us; hence we should, in acquiring knowledge, ignore sense impressions and develop reason. In codifying such logical reasoning, Aristotle (384–322 BC) set down rules of inference and recognized the importance of axioms for logic, postulates for the subject at hand, definitions of terms, and the importance of giving logical arguments that start with the postulates. By combining Aristotle's precise formulation of logic with Thales's method, the main elements of modern science were then in place. Most philosophic analyses of the philosophy of Thales come from Aristotle. Thales is credited as the first person about whom we know to propose explanations of natural phenomena that were materialistic rather than mythological or theological. Because his views of nature gave no role to mythical beings, Thales's theories could be refuted by evidence. Arguments could be put forward in attempts to discredit them. Thales's hypotheses were rational and scientific.

The Model of Hierarchical Complexity (MHC) follows in that tradition (see "Presenting the Formal Theory of Hierarchical Complexity," this issue). The MHC is a mathematical theory of the ideal. It is a perfect form as Plato would have described it. It is like a circle. A circle is an ideal form that exists. Once one draws a circle, something additional and different has been created. The new creation is a *representation* of a circle, but it is not, itself, a perfect ideal circle. The lines have width whereas a circle does not, and thus cannot perfectly represent the perfect form itself. The representation is not perfect nor can a drawn circle be perfectly round. This distinction between the ideal form and representations of the ideal is important for understanding the MHC and its relationship to stage of performance.

Historically, there were three further developments necessary before this Model would be possible to develop. The first was the success of Copernicus (1992) in showing that the sun is the center of the solar system. By using mathematics to represent the orbits of the planets in some sense we could say that this was the first mathematical model. Second, modern thinking about the brain and behavior began with the French philosopher René Descartes (1596–1650). According to Descartes (1954), all action is a response to an event. He thereby introduced the notion of the stimulus and the response. Descartes suggested that "animal spirits" flowing through the nerves of animals or humans served a function similar to stimuli in relation to automatic behavioral responses, that is, reflexes in humans and animals. The term reflex came from the notion that the flow of animal spirits produced by a stimulus was somehow reflected by the brain into an outgoing flow that eventually produced some behavior. Later, G. T. Fechner (1966) laid the basis for the application of the experimental method to psychology. He established psychophysics by introducing psychophysical scales and showed how to relate psychological variables to stimulus variables. This is what the Model of Hierarchical Complexity does. It relates stage of performance to the order of hierarchical complexity of tasks. Lastly, in the early 1960s, many others' work (e.g., Krantz, Luce, Suppes, and Tversky, 1971; Suppes, Krantz, Luce, and Tversky, 1989; Luce, Krantz, Suppes, and Tversky, 1990). introduced the representational theory of measurement. It is the basis for the Model of Hierarchical Complexity.

APPLICATIONS OF THE MODEL

Problem-solving tasks of various kinds have been ordered for animals using the MHC and for humans using the MHC and the closely related Skill Theory of Fischer (1980). Because animals cannot perform tasks at the postformal orders (but see "Toward a Cross-Species Measure of General Intelligence," this issue), they are not included in Table 3's list of applications. Researchers using the content-free scoring of stages of human performance have not found differences between males and females.

POSTFORMAL THOUGHT IN THE MODEL OF HIERARCHICAL COMPLEXITY

As indicated in Table 1, the tenth order of hierarchical complexity is named *formal*. In settings with an effective educational system for adolescents, most students without learning disabilities become able to perform at this stage in at least some areas (e.g., in some courses of study in school). For many years after Piaget's work in the 1950s, he and others assumed that this stage category called *formal operations* was the stage at which human development reached its highest plateau. In the last quarter of the 20th century, researchers were identifying more complex activities than could be performed using formal logic. Some have shown (e.g., Commons and Richards, 2002) that Piaget himself had to employ postformal reasoning in order to develop his system to define formal operations. These more complex activities were soon grouped into the category, *postformal* operations. This was understandable, because it took many years of analysis to distinguish that there were stage differences among various postformal activities. The term continues to apply generically to stages of development that are more hierarchically complex than formal operations. There are four such stages: Systematic, Metasystematic, Paradigmatic, and Cross-paradigmatic. These are numbered 11 through 14 (Table 1). These stages and their significance are discussed in the following article.

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