World Futures, 64: 361–374, 2008

ISSN 0260-4027 print / 1556-1844 online
DOI: 10.1080/02604020802301196

FRACTAL TRANSITION STEPS TO FRACTAL STAGES: THE DYNAMICS OF EVOLUTION, II

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Successful applications of hierarchical complexity to the behaviors of organisms, animals and humans, and social entities evidence the scaling properties of self-similarity, thus the bounded fractal characteristics of orders of hierarchical complexity. The theory specifies an identical sequence of discrete-state transition steps required from each stage of performance to the next. It repeats at all scales. Tasks nested within the step sequence evidence self-similarity with the orders of complexity. This model introduces questions about noise categories when system tasks are fully accounted for, dependent, self-similar, and measurable. Ubiquitous transition steps are inherent dynamics of evolution.

KEYWORDS: Dynamics, evolution, fractal, hierarchical complexity, nonlinear, noise, scaling, self-similar, stage, transition steps.

The *how* of development and many evolutionary dynamics are explained by transition steps described by the Model of Hierarchical Complexity. The focus in this article is on that *how*: the dynamic relationships and detailed patterns comprising hierarchical complexity. These derive from the empirically established sequence of transition steps, *required* from any order of hierarchical complexity to the next (Commons and Miller, 1998; Commons and Richards, 2002; Ross, 2007a). The steps in transition help one get to the heart of the hierarchical complexity view of evolutionary dynamics: if any behavior is evolving in any domain, it is because it is moving through the transition steps in that domain. For the steps and stages of hierarchical complexity to thus apply to all behaviors, by deduction they are fractal, although bounded. The purpose here is to shine detailed light on that inherently fractal nature. Sequenced to follow the "high level" discussion of the dynamics of selectionism and stage change in evolution, here, details are offered in a systematized fashion. The hope is to facilitate "seeing" hierarchical complexity dynamics at widely diverse scales. These scales include time, space, and size in entities that demonstrate behaviors that range from the microscopic to human societies.

To present the fractal nature of stages and transition steps in a methodical fashion, this article is organized as follows. To situate the discussion of transition

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steps it begins with a discussion of key features of the Model introduced earlier in this issue. The central section explicates the sequence of transition steps as it occurs between less complex and more complex orders of hierarchical complexity. This reveals the fractal nature of that sequence and how its nested dynamics reveal a deeper fractal nature of the overall Model. Previously analyzed data are presented to explicate this. Research questions derived from the hypothesis are broached. The concluding discussion indicates a range of implications suggested by the fractal nature of hierarchical complexity in general and in particular, the bounded fractal nature of the transition steps.

KEY FEATURES OF THE MODEL: THE FRACTAL NATURE OF NONARBITRARY HIERARCHICAL COORDINATION

The Model of Hierarchical Complexity (see "Presenting the Formal Theory of Hierarchical Complexity," this issue; Commons, Goodheart, Pekker, et al., 2007; Commons, Trudeau, Stein, Richards, and Krause, 1998) offers a standard method to examine certain universal patterns in a system's process of developing hierarchical increases in the complexity of tasks it performs. It applies across a broad range of events and scales of events. It applies to any events or occasions in which information is organized. Entities that organize information include organisms (e.g., cells, humans, other animals), groups and social systems, and machines. The reason the Model applies so broadly is that it is a general theory with a mathematical method of measuring tasks that can contain any kind of information. This results in a quantal notion of stage of task performance. Its use of purely quantitative principles makes its concepts, such as stage and steps, universally applicable to any context where action happens.

Nonarbitrary Coordination

A stage of any given order of hierarchical complexity is formed by coordinating two or more task-actions at the preceding, lower order in a nonarbitrary way. This section unpacks that notion with examples to further explicate stages and lay groundwork for discussing transition steps later. At the Sentential stage 5 (cell i, Figure 1), one can form sentences for the first time, and also begin to use pronouns.[1] To form a sentence first requires at least a noun (let us say, cell g) and a verb (let us say, cell h). These are task performances acquired at Nominal stage 4. At the Nominal stage, verbs and nouns are used for the first time, as one learns to relate concepts and name things (e.g., the story of blind and deaf Helen Keller's relating the concept of water to the actual substance of water and speaking the name for the first time). This example explicates the axiom that a task at any order of complexity (in this case, Sentential order 5) is formed by coordinating at least two task-actions of the preceding order in a nonarbitrary way (uses of Nominal order 4 noun and Nominal order 4 verb, in this case).

To explicate the notion of nonarbitrary coordination of tasks, we can imagine that after Helen Keller named the water running from the pump that first time, she later formed some simple Sentential stage 5 sentences about it. They might

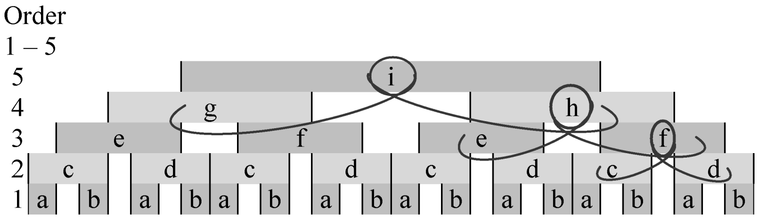


Figure 1. Representation of hierarchical complexity's nonarbitrary, nonlinear coordination of lower-order task-actions. The self-similar looping symbols—"larger" at each order of greater hierarchical complexity—represent the coordination activity performed in all cases of organizing information (coordinating task-actions) at the next higher order. Copyright © 2007–2008 by Sara N. Ross. Reproduced with permission.

have been "Water runs," or "Pump water." If the nouns and verbs in either of these sentences were not coordinated properly, the sentences might have entirely different meanings, for example, "Runs water" or "Water pump." To successfully perform the task of describing what the water does, "Water runs" is the required, nonarbitrary coordination of the Nominal stage 4 named concepts. To ask someone to pump the water in the English language, "Pump water" rather than "Water pump" is the required, nonarbitrary coordination. This same pattern or method of developing increased hierarchical complexity from order to order extends to the last known stage. At each stage, *at least* but *not necessarily only* two lower-staged actions or tasks are coordinated in a nonarbitrary way.

The Fractal Nature of the Stages of Hierarchical Complexity

One major implication of this universal, self-similar pattern that shows up at all scales of tasks of any kind is, by definition, that the stages described by the Model of Hierarchical Complexity are fractal. This is important for theoretical, analytical, and practical reasons—many of which are introduced throughout this special issue—and also for understanding the transition step sequence between each stage and its implications. Examples of different kinds of scales at two different stages of hierarchical complexity illustrate this "scaling property" of the stages.

1. Examples of Sensory or Motor stage 1 actions at different physical scales (see Commons and Miller, 2007 for discussion)
    a. The cellular protein called the potassium channel in the corneal endothelium opens and closes like a reflex switch when a potassium ion passes through it and thereby produces a tiny electrical current (Liebovitch and Shehadeh, 2005).
    b. The mollusk opens its shell when surrounding water moves. Reflexively, if something touches its membrane, it closes the shell.
    c. The newborn animal moves its limbs, eyes, head, or lips.

2. Examples of Formal stage 10 actions at different time scales
    a. To choose a different action: “If I keep waiting for this traffic jam to clear, I will miss my appointment. If I get off the highway at this next exit, I can drive through the city and may not be too late.”
    b. To perform an administrative procedure: Before authorizing the company’s payment of a supplier’s invoice, a clerk examines the shipment’s packing slip for evidence of authorized personnel’s validation of quantities received, and compares the packing slip to the original purchase order’s type, and quantity details. The authorized price listed on the purchase order is compared to the invoiced price. The invoiced calculation of payment due is verified. If all evidence supports the accuracy of the supplier’s invoice, it is marked as approved for payment.
    c. To design a curriculum or assembly line: The required outcomes are identified. All materials or components and the sequence and means of using them to produce outcomes are specified. Procedures to measure quality and deal with low quality outcomes are specified.
    d. To ensure the rule of law in a society is administered objectively: Establish an independent judiciary.

These examples indicate that tasks of the same order of hierarchical complexity take perhaps unlimited different forms at different scales of time, space, and task-performers. How the performance of a task at any fractal stage of hierarchical complexity may change to the performance of a task at the next higher stage is addressed by the sequence of discrete transition steps.

EXPLICATING THE TRANSITION STEP SEQUENCE

Stages of hierarchical complexity explain how development and evolution come about at any scale. A full and precise explanation must include the transition step sequence. This sequence accounts for how the stages themselves come about. Reflection on the Helen Keller example reveals that the example was *silent about how* the Nominal stage 4 noun and verb elements would be selected and coordinated in the context of whatever situation would trigger utterances of sentences. The example indicated only *what* lower stage tasks had to be coordinated in a nonarbitrary way to perform the next stage task of forming a sentence. The transition step sequence answers the *how* question.

Just as the orders of hierarchical complexity fall on an ordinal scale, so also do the transition steps that individually comprise the transition sequence. One way to visualize the relation of the transition step ordinal scale to the orders of hierarchical complexity is as follows. The orders of increasing hierarchical complexity fall on one ordinal scale (i.e., 0, 1, 2, 3, 4 . . . 14). The transition steps that lead from one order to another fall on another ordinal scale, which runs from 1 through 8. Figure 2 represents the relation of the two ordinal scales, with the step sequence repeated and aligned vertically over the horizontally aligned orders. The unequal spacing in Figure 2 is a visual indicator of the ordinality of the scales. The ordinal nature

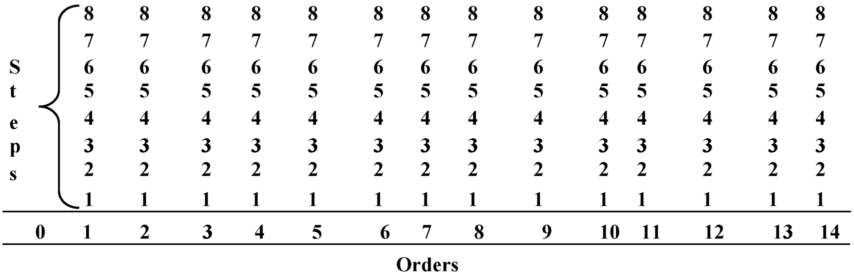
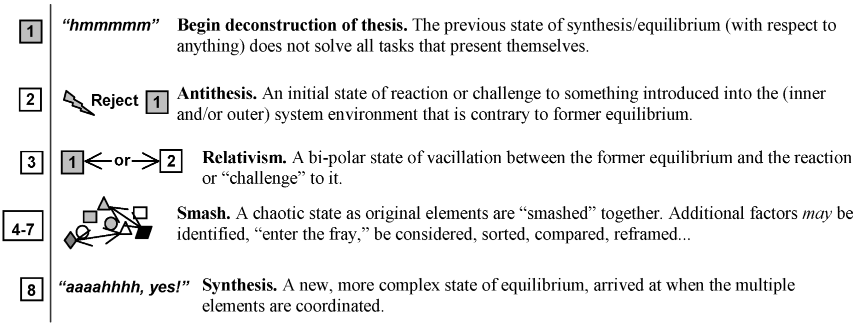


Figure 2. The unequally spaced ordinal scales of orders and transition steps. Copyright © 2007 by Sara N. Ross. Reproduced with permission.

means these are not like degrees of temperature that are on an equally spaced scale. Ordinal scales are simple counts of occurrences; in this case, task-actions.

The stages of hierarchical complexity are the axiomatically defined, mathematically specified performances of tasks. The empirically based transition steps' dynamics are not yet mathematically specified (although they are partially described by signal detection theory). Transition step dynamics characterize task-performing complex adaptive systems when they are not operating in a hypothetically "perfect" stage–based equilibrium. In other words, they are ubiquitous. Humans, for example, spend much of their time in transitions at various scales of activity in their multiple domains of life. In contrast to the dry step descriptions we detail elsewhere in this issue, Figure 3 offers a version of the transition step sequence that may help correlate the steps to diverse, first-hand human experiences.

Adaptation characterizes the transition steps, and is the process by which changes in stage come about. The steps describe the *process* of adapting: learning to perform tasks at the next order of hierarchical complexity. Commons and Miller (1998) and Commons and Richards (2002) have described these transitions and, adding to the preceding article on selectionism and stage change (this issue), offered reasons why transition does or does not happen. To summarize



Begin deconstruction of thesis. The previous state of synthesis/equilibrium (with respect to anything) does not solve all tasks that present themselves.

Antithesis. An initial state of reaction or challenge to something introduced into the (inner and/or outer) system environment that is contrary to former equilibrium.

Relativism. A bi-polar state of vacillation between the former equilibrium and the reaction or "challenge" to it.

Smash. A chaotic state as original elements are "smashed" together. Additional factors *may* be identified, "enter the fray," be considered, sorted, compared, reframed...

Synthesis. A new, more complex state of equilibrium, arrived at when the multiple elements are coordinated.

Figure 3. Anthropomorphized rendition of the transition step sequence. Copyright © 2006–2008 by Sara N. Ross. Reproduced with permission.

how it happens,[2] steps 1 through 3 are activities that "deconstruct" whatever successfully performed task had previously afforded a sense of equilibrium in the environment. Equilibrium is supported by reinforcement. These initial steps ensue in the face of a drop in perceived reinforcement to continue the previous task behavior (Commons and Richards, 2002). The real or perceived loss of reinforcement may initiate the learning opportunities of the steps. These are opportunities because the performance of many of the transition tasks involves trying new tasks that do not mimic prior behavior. Some of the new actions result in new reinforcement, some may not. Such behavioral cusps have "consequences for the organism beyond the change itself, some of which may be considered important" (Rosales-Ruiz and Baer, 1997, p. 537, as quoted in Commons and Richards 2002, p. 160; hierarchical complexity and the chaos theory study of such cusps may develop further insights into such consequences). As the "reconstructive" transition steps 4 through 7 are performed, the organism is experimenting with new options in a creative, chaotic fashion. When incorrect options are eventually discarded and a successful coordination of lower-stage tasks takes place at step 8, the new synthesis at the next higher stage results in an equilibrium for that new, higher-staged task performance. Each step involves adaptive learning, regardless of the nature of the task-performing organism. The following list is an abbreviated description of the eight ordinally scaled transition steps.

1. Reinforcement of thesis decreases
2. Antithesis: Negation or complementation
3. Relativism: Alternation of thesis and antithesis
4. Smash_0: Synthesis begin
5. Random hits, false alarms, and misses, low correct rejections (Smash_1)
6. More hits, low misses and correct rejections excess false alarms, (Smash_2)
7. Correct rejections and excess misses, low hits and false alarms (Smash_3)
8. Synthesis and new thesis: New temporary equilibrium

In humans, one of the easiest areas to see a snapshot of transition step dynamics is in a decision-making process that happens in a brief time window. Table 1 presents transcripted data, scored in transition steps with bracketed scoring comments, gathered in a recent study (Ross, 2007b; Ross & Commons, 2007).

THE FRACTAL NATURE OF THE TRANSITION STEPS

The fractal model of steps in transition (Ross, 2007b; Ross and Commons, 2007) extends and deepens the Model of Hierarchical Complexity and invites applications to other developmental science models, decision theory, time series analyses, agent modeling and game theory, and policy and other analyses. It is a further explication of the nonlinearity of tasks that characterize the Model. Because the generic pattern of transition steps that transpires between each order of hierarchical complexity is identical, by definition it is a recurring, self-similar pattern.

Table 1
Scored Transition Steps in a Personal Decision-Making Process

Score	Speaker's Narrative
Stage 10	I need to get the grass mowed. With all the rain we've had, it's getting too long. By the time I get back from the trip, it'll be even longer. So I have to get it mowed before I go, and need to figure out how to get that done. [thesis: I need to schedule the time to do it.]
Stage 10 Step 1	I have to put too much time in on this paper, I can't afford to take time out to mow, even though it would it do me good. So, I've got 1, 2, 3, 4, 5 days before I leave. I don't know what the weather report is. [begins deconstructing thesis of speaker doing it.]
Stage 10 Step 2	So, maybe I can ask N——— to do at least the flatter parts and get part of it done. [antithesis: N——— do it]
Stage 10 Step 3	Maybe I can hire M——— to do some of it, but he's not comfortable mowing the hills, either. [relativism: speaker do it or N———/M———]
Stage 10 Step 4	[begins smash$_0$] It'll take twice as long to get it done if I don't get it while it's shorter. If I squeeze in time to do it, I'm afraid . . .
Stage 10 Step 5	[begins smash$_1$] Well, if I give M——— really clear instructions, maybe even if I give N——— really clear instructions, they can take the time to rake all that excess. And the poor mower, damn, it's not behaving very robust anymore, so I don't how it's going to do. And they're not as careful as I am. So if I have N——— do it, um, he could get a few hours in, but not much this weekend with working. Even with M———, it'd probably be early in the week. And then of course having him around is always distracting for getting my work done, cause he's always interrupting, the mental distractions of having him around. It's hard to concentrate when he's around, and that's what I mostly need to do right now. Also, if I have N——— do it, I don't have to pay him anything. But if I have M——— do it, I'll have to pay him, and I can't see paying 10 bucks an hour for just mowing. Can't afford to let it go, it's really bad, at least parts of it.
Stage 10 Step 6	[begins smash$_2$] Of course the parts that are worst aren't the parts next to the street. Could say time expands with the needs, and see if I get enough done on this paper that I could go ahead and do the worst part. Have N——— do some, skip M———, at least get the most cosmetic needs met, and then deal with the rest when I get back. Do I have it resolved? Um . . . not really, I'm not really sure.
Stage 10 Step 7	[begins smash$_3$] I want to get the grass mowed without me doing it. This is a simple problem to solve. I should find out how much N——— can commit to doing, and then if that leaves only a half hour or an hour's worth of mowing for me to do, I can make that up, take it out of my sleep, that's no big deal. So I'll skip M———: that's much ado about nothing, to hire somebody at this point. And, uh, deal with the front hill when I get back. It can wait longer, it's just horse pasture.
Stage 10 Step 8 Equals New Stage 11	[Synthesis/new thesis] So that's it. I'll see how many hours I can get N——— to do, and I'll fill in the tougher places myself if I need to, and just not worry about it. It's not a big problem, just societal pressure to keep the grass low and of course now the mechanical pressure of the mower. It seems to be on its last legs, and I can't afford to replace the tractor. So, that's what I'll do.

There are two specific, seemingly different yet intimately related ways that the fractal nature of the transition steps is evident. As indicated earlier, the Model of Hierarchical Complexity describes fractal stages of tasks. Regardless of the scale of tasks performed at a given stage, the transition step sequence accounts for the tasks' possibility of occurrence at all. Because the transitions transpire universally at different scales of time, space, and task-performer, they—and the stages—demonstrate scaling properties. The sequence of steps occurs in self-similar fashion regardless of the scale of tasks. Thus, by definition, the transition steps and the sequence in which they occur are fractal. This is the first, most readily evident way to determine their fractal nature.

Consistent with fractal characteristics, there is a deeper layer of hierarchical complexity's fractal nature evident within the sequence of steps. An explanation is useful before illustrating how this appears to date. Given the building-block structure of hierarchical complexity (Figure 1), at each stage a task-performer has accrued a greater number of possible stages of performance, a repertoire from which to select how to accomplish tasks. Depending on the complexity of the task at hand, there may be multiple tasks at different stages that have to be selected and coordinated before the overall task can be accomplished. That is, more elements of different-staged tasks, like subroutines, may be necessary to solve a particular task.

This is particularly true for humans and their various levels of social groupings and organizations whose efforts to perform real world tasks are enabled by symbols, representations, and language with complex concepts. Many such elements may complicate the transition from tasks at one stage to another. Tasks may include, for example, a more complex physical response, more complex decision making, more complex problem-finding and problem-solving analyses, or combinations thereof. If the task requires higher-stage performance than the person or social entity has acquired in that domain, then successful performance requires transition steps to increase the hierarchical complexity to perform the higher-order action.

Due to such complexity at higher stages, the transition steps' tasks often involve subtasks. The subtasks lead deeper into bounded fractal territory. The more complex the overall task, the greater the quantity of nested subtasks that are likely to be components of it. A possible resulting pattern of nested hierarchical complexity is indicated in Figure 4, where *n* indicates a stage of performance in the task's

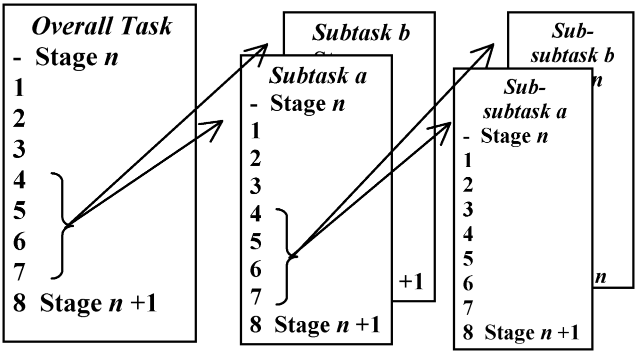


Figure 4. Representation of the fractal nature of transition steps' subtasks. Copyright © 2007–2008 by Sara N. Ross. Reproduced with permission.

domain. Ordinal step scores are used in that figure instead of text, referring to transition tasks at any scale.

In initial analyses performed to date (Ross, 2007b; Ross and Commons, 2007), the deeper fractal nature of the overall Model is indicated in transition step dynamics appearing at Abstract stage 9 to Metasystematic stage 12 thought. Subtasks such as those represented in Figure 4 have been identified, beginning at these stages and transitioning from them. Examples are provided and discussed in what follows to illustrate this pattern.

To score the hierarchical complexity of tasks requires a starting point. This is like performing any other task: we must know where to begin. When scoring a narrative, there is a beginning proposition given by the speaker or writer that indicates the beginning of a task (Commons, Rodriguez, et al. 2007). The proposition may be in the form of a thesis statement, a decision that has been framed, or a question that is posed. If subtasks are part of the person's actions to complete the overall task, such subtasks appear in the smash steps of the transition sequence. That is the locus of subtask propositions. These are nested within the overall task (Figure 4). The more of these there are, the more complex the overall task is for the person to perform.

The traditional approach to scoring transitions (Table 1) may be augmented by more fine-grained scoring analyses of the smash transition steps. It was through such fine-grained analyses that more of the Model's fractal nature became evident. The fine-grained analysis is called the "thesis-nests" approach. When the narrative reported in Table 1 is scored using the thesis-nests approach, the subtasks illustrated in Figure 5 appear. Thesis-nests-scored transcripts to date indicate that

<table>
<tr><th colspan="6">Stage-Step Scores of Thesis-Nests
And Sub-subtasks</th></tr>
<tr><th>Thesis 1</th><th>Thesis 2</th><th>Thesis 3</th><th>Thesis 4</th><th>Thesis 5</th><th>Synthesis</th></tr>
<tr><td>9-</td><td>9-</td><td>9-</td><td>9-
9-2</td><td>10-4</td><td>10-3</td></tr>
<tr><td rowspan="2">9-</td><td rowspan="2">9-1</td><td colspan="2">Sub-subtask nested thesis</td><td rowspan="2">10-7</td><td rowspan="2"></td></tr>
<tr><td colspan="2">10-5</td></tr>
<tr><td>9-</td><td></td><td colspan="2">10-5</td><td></td><td>10-7</td></tr>
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</table>

Figure 5. Skeleton overview of Table 1 narrative's thesis-nests scoring.

Table 2
11-Minute Decision-Making Trial: Thesis-Nests Scored for Stage-Step

Thesis 1	Thesis 2	Thesis 3	Thesis 4	Thesis 5	Thesis 6	Thesis 7	Thesis 8	Thesis 9	Thesis 10
10-1	10-1	11-	10-1	10-1	12-1	11-	11-	10.0	10-1
10-2	10-2	11-1	10-2	11-	12-1	11-	11-1	10-1	10-2
10-3	10-3	11-2	10-3	11-	12-2		11-2	10-2	10-3
10-3	10-3	11-2	10-4	11-1			11-3		10-3
10-3	10-4	11-3	10-5				11-4		10-4
10-4	10-4		10-5				11-5		11-
10-5	10-5		10-6						11-
10-6	10-7		10-6						11-
10-7									11-1
11-									11-2
									11-2

theses do not necessarily develop in a linear chronological sequence (and of course, when relying on audio recordings, any transition steps that occur in silent thought are not possible to capture and indicate). The stages in Figure 5 involve Abstract 9, Formal 10, and Systematic 11. Transition step ordinals, if any, are shown following the hyphen after a stage ordinal (e.g., 10–2 represents Formal stage 10, step 2 in transition).

Nested theses in transitions may extend to only one layer of subtasks, without such nested sub-subtasks, as indicated earlier. Scored transition data in a more complex decision process by a single subject reported by Ross (2007b) and Ross and Commons (2007) indicated twice as many thesis-nests and no further nesting of sub-subtasks (Table 2). The fractals of the overall Model appear more clearly in this example, with some theses progressing through at least one full stage (Theses 1 and 10) or more than half of the transition steps to the next stage (Theses 2, 4, and 8).

Subtasks embedded in and thereby constituting transition steps may serve as further stimuli (e.g., new information) in the process and seem to be prerequisites before the overall task can be accomplished. The subtasks themselves may or may not be comprised of further subtasks, as illustrated earlier. Thus, depending on the complexity of the overall task, transition steps at varying stages of hierarchical complexity can extend into increasingly fine-grained tasks with transition steps that are themselves fractals of stages described in the Model of Hierarchical Complexity.

Calculation of the fractal dimension is performed after data are plotted in a dispersion analysis, onto which a linear regression line is plotted. The fractal dimension indicates an inverse power-law scaling relation, and equals 1 – slope, where slope refers to the log-log regression line (Holden, 2005). Ross (2007b) and Ross and Commons (2007) reported that Table 2's data returned a 1.66 when thus calculating the fractal dimension. That value falls above 1.5 and into the range called white noise. The 1.66 would thus indicate *non* fractal dynamics. By contrast, when the transcript was scored in the traditional chronological sequence (as illustrated in Table 1 for a different trial), the calculated fractal dimension was in the 1.04 noise range. While 1–1.5 can indicate pink noise (Holden, 2005), this low measure is typically considered brown noise (Van Orden, email communication, March 21, 2008). Both brown and pink noise indicate positive correlations over short and long terms, respectively (Aks, 2005). Pink noise is the one recognized as indicating *fractal* dynamics. Pink noise indicates positively correlated, dependent behaviors, and white noise indicates uncorrelated, independent behaviors (Aks, 2005; Holden, 2005). Regardless of these traditional categories, all steps in transition are by definition dependent, and subtasks appear interdependent.

The same system (the participant in the study) generated data that returned two different values when calculating the fractal dimension. The 1.66 measure, which is generally assumed to indicate a nonfractal behavior range, resulted from allowing the hierarchical complexity measures to "zoom in on" the transition step dynamics. Yet the model in nonlinear science about pink noise/fractal and brown and white noise/nonfractal measurement ranges does not fit these data. Questions and possible implications are suggested in what follows.

DO THE MEASURABLE TRANSITION STEP TASKS GENERATE BROWN, PINK, AND WHITE NOISE?

The Model of Hierarchical Complexity is a general theory of tasks. Tasks comprise behaviors. The stages and transition steps account for all the tasks performed by a system at any scale. All tasks are definable and measurable. By definition, stages and transition steps are dependent and self-similar. We stand on firm ground in that regard. Yet we are in the very earliest stages of applying nonlinear methods to explicate the fractal characteristics of hierarchical complexity's dynamics, and finding people to work with. Ross is yet a novice in applying the methods. It is not yet clear if current methods are designed to accommodate all of the fractal dynamics of transitions, or if different models must be found to fit transition data. For example, does the thesis-nest scoring, which unambiguously evidences bounded fractals of the orders of complexity, equate to a form of data shuffling that results in brown and white noise indicators when run through traditional methods? Despite these early questions, to move the discourse along, we tender the following hypothesis and research questions. These serve as place markers for further discourse with scientific communities concerned with nonlinear dynamics, and in particular on some of the implications of the Model of Hierarchical Complexity, what it identifies and measures, and its fractal nature.

1. Interdependency and scaling self-similarity are classic requirements in defining fractals; the transition step sequence indicates not only high but also requisite degrees of both characteristics in dynamic system tasks at any scale from one stage to another as well as from one step to another between the stages.
2. Calculated fractal dimensions have pertained to brown and pink noise, with pink noise considered fractal. As indicated earlier, fractal transition step dynamics

have measured in the both the brown and white noise ranges. By deduction, it appears nonlinear methods may categorize them erroneously as non fractal. What methods fit hierarchical complexity data? Might research that measures system dynamics using hierarchical complexity re-frame assumptions about some characteristics of noise in system behaviors?

3. The assumption to date in nonlinear science has been that there is no way to measure the ubiquitous $1/f$ pink noise. If hierarchical complexity is a measure that can be applied to the task actions of systems producing $1/f$, will further hierarchical complexity data affirm the immeasurability of $1/f$? If measurements are assigned to the original dynamics that give rise to noise, that is, the tasks being performed, does this capability inform classifying pink noise?

A FEW IMPLICATIONS FOR APPLICATIONS

This presentation indicates the Model responds to the suggestions that "the development of well-specified models of the principles or mechanisms of human cognition giving rise to $1/f$ noise is long overdue" (Farrell, Wagenmakers, and Ratcliff, 2006, p. 737), including the need for understandings of embodied cognition and finding appropriate tools to study globally emergent phenomena (Van Orden, Holden, and Turvey, 2003). It suggests that using the Model to measure the tasks that give rise to $1/f$ noise may inform the debate over the presence of $1/f$ noise in human cognition, for example, ubiquitous or not, including whether it indicates fractal processes (e.g., Farrell, Wagenmakers, and Ratcliff, 2006; Thorton and Gilden, 2005) and where and why they occur. Farrell et al. have been concerned that "previous investigations . . . of serial correlation in psychology have fit only a single model, a fractal model, and have not considered alternative models of the fluctuations in psychological series" (p. 738). They argue that $1/f$ noise cannot be considered in isolation. Likewise, Amunategui and Dowd (2006, p. 284) have argued that "we need an analytic tool capable of discriminating between random noise and nonlinear determinism." To date, perhaps data sources have been conceived in a too-limited way, for example, "It makes sense to start with variables that produce nearly continuous data streams. . . . This points to events that produce continuous streams of finite and clearly demarcated cycles. In humans, this is narrowed down to two signals, respiratory and cardiac cycles" (Amunategui and Dowd, 2006, p. 286). Yet, the transition step sequence indicates continuous streams of a greater number of clearly demarcated, more complex cycles, with finiteness apparent when the scales of attention are adjusted to accommodate them. Might long-term time series studies, for example, Delignières, Fortes, and Ninot (2004), become able to account for what happens between the variables specified?

It appears that hierarchical complexity considerably expands and potentially accelerates the study of phenomena in far more of their complexity. When the Model is employed to examine social phenomena as suggested elsewhere in this special issue, implications for other domains of theory, policy, and practice should become apparent.

THE FRACTAL DYNAMICS OF EVOLUTION

This article described how the stages of hierarchical complexity and the transition step sequence occur at different scales of time, space, and task-performer, and indicated the complexity and variety of measurable task performances. Because the generic pattern of transition steps that transpires between each stage of hierarchical complexity is identical, by definition it is a recurring, self-similar pattern. Its fractal characteristic was illustrated through discussion and diverse examples. Each stage and each transition step sequence is quantitatively and qualitatively different depending on a system's inherent complexity, the order of hierarchical complexity of the task, and whether transitions occur during the time behavior is observed. This pattern reflects the dialectical nonlinear processes that characterize evolution's dynamics: increases in hierarchical complexity made possible by transition dynamics. Research questions are proposed to place attention both on methodology and on implications of hierarchical complexity measurements of all tasks performed by all systems that perform tasks.

NOTES

1. For non-language examples of the stages discussed here, and for higher stage examples of nonarbitrary coordinations, see other articles in this special issue.
2. See the transition step table in "Introduction to the Model of Hierarchical Complexity," this issue.

REFERENCES

Aks, D. J. 2005. 1/f dynamic in complex visual search: Evidence for self-organized criticality in human perception. In *Tutorials in contemporary nonlinear methods for the behavioral sciences,* Eds. Riley, M. A., and Van Orden,G. C., 319–352. www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp (Accessed 2 May 2007.)

Amunategui, L. F., and Dowd, E. T. 2006. Autopoiesis and nonlinear modeling methods: An empirical approach to psychotherapy process research. *Nonlinear Dynamics, Psychology, and Life Sciences* 10(2): 275–300.

Commons, M. L., Goodheart, E. A., Pekker, A., Dawson, T. L., Draney, K., and Adams K. M. 2007. Using Rasch scaled stage scores to validate orders of hierarchical complexity of balance beam task sequences. In *Rasch Measurement: Advanced and specialized applications*, Eds. Smith, E. V. Jr., and Smith, R. M., 121–147. Maple Grove, MN: JAM Press.

———, and Miller, P. M. 1998. A quantitative behavior-analytic theory of development. *Mexican Journal of Experimental Analysis of Behavior* 24: 153–180.

———, and Miller, P. M. 2007. What animals never operantly coordinate actions with sensory input: The earliest stages of animal behavior. Paper presented at the international conference on comparative cognition, March 24–27, in Melbourne Beach, FL.

———, and Richards, F. A. 2002. Organizing components in combinations: How stage transition works. *Journal of Adult Development* 9(3): 159–177.

———, Rodriguez, J. A., Miller, P. M., Ross, S. N., Lo Cicero, A., Goodheart, E. A., and Danaher-Gilpin, D. 2007. *Applying the model of hierarchical complexity*. Unpublished manual, Dare Institute. Available at commons@tiac.net

———, Trudeau, E. J., Stein, S. A., Richards, F. A., and Krause, S. R. 1998. The existence of developmental stages as shown by the hierarchical complexity of tasks. *Developmental Review* 8(3): 237–278.

Delignières, D., Fortes, M., and Ninot, G. 2004. The fractal dynamics of self-esteem and physical self. *Nonlinear Dynamics in Psychology and Life Sciences* 8: 479–510.

Farrell, S., Wagenmakers, E. J., and Ratcliff, R. 2006. $1/f$ noise in human cognition: Is it ubiquitous, and what does it mean? *Psychonomic Bulletin and Review* 13(4): 737–741.

Holden, J. G. 2005. Gauging the fractal dimension of response times from cognitive tasks. In *Tutorials in contemporary nonlinear methods for the behavioral sciences,* Eds. Riley, M. A., and Van Orden, G. C., 267–318. www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp (Accessed 2 May 2007.)

Liebovitch, L. S., and Shehadeh, L. A. 2005. Introduction to fractals. In *Tutorials in contemporary nonlinear methods for the behavioral sciences*, Eds. Riley, M. A., and Van Orden, G. C., 178–266. www.nsf.gov/sbe/bcs/pac/nmbs/nmbs.jsp (Accessed 2 May 2007.)

Ross, S. N. 2007a. Effects of a structured public issues discourse method on the complexity of citizens' reasoning and local political development. *Dissertation Abstracts International*, B 68(02), UMI No. 3251492.

———. 2007b. The fractal nature of stage change: A model and transition data. Poster presented at the annual meeting of Society for Mathematical Psychology, July 25–28, in Costa Mesa, CA.

———, and M. L. Commons. 2007. *The fractal nature of stage change: A model and transition data*. Paper presented in theoretical foundations of nonlinear dynamics at the annual international conference of the Society for Chaos Theory in Psychology and Life Sciences, July 27–29, in Orange, CA.

Thorton, T. L., and Gilden, D. L. 2005. Provenance of correlations in psychological data. *Psychonomic Bulletin and Review* 12(3): 409–411.

Van Orden, G. C., Holden, J. G., and Turvey, M. T. 2003. Self-organization of cognitive performance. *Journal of Experimental Psychology* 132: 331–350.